SOLUTIONS FOR CHAPTER 21
PROBLEMS 4, 12, 19, 25, 33, 40, 50, 75, 89, 96.

21.4. IDENTIFY: Use the mass \( m \) of the ring and the atomic mass \( M \) of gold to calculate the number of gold atoms. Each atom has 79 protons and an equal number of electrons.

SET UP: \( N_A = 6.02 \times 10^{23} \text{ atoms/mol} \). A proton has charge \( +e \).

EXECUTE: The mass of gold is 17.7 g and the atomic weight of gold is 197 g/mol. So the number of atoms is \( N = \frac{17.7 \text{ g}}{197 \text{ g/mol}} \times 6.02 \times 10^{23} \text{ atoms/mol} = 5.41 \times 10^{22} \text{ atoms} \). The number of protons is 

\[ n_p = (79 \text{ protons/atom})(5.41 \times 10^{22} \text{ atoms}) = 4.27 \times 10^{24} \text{ protons} \].

\( Q = (n_e)(1.60 \times 10^{-19} \text{ C/proton}) = 6.83 \times 10^5 \text{ C} \).

(b) The number of electrons is \( n_e = n_p = 4.27 \times 10^{24} \).

EVALUATE: The total amount of positive charge in the ring is very large, but there is an equal amount of negative charge.


SET UP: Like charges repel and unlike charges attract.

EXECUTE: (a) \[ F = \frac{1}{4\pi \varepsilon_0} \frac{|q_1 q_2|}{r^2} \]. This gives \( 0.200 \text{ N} = \frac{1}{4\pi \varepsilon_0} \frac{(0.550 \times 10^{-6} \text{ C})|q_2|}{(0.30 \text{ m})^2} \)

\[ |q_2| = +3.64 \times 10^{-6} \text{ C} \]. The force is attractive and \( q_1 < 0 \), so \( q_2 = +3.64 \times 10^{-6} \text{ C} \).

(b) \( F = 0.200 \text{ N} \). The force is attractive, so is downward.

EVALUATE: The forces between the two charges obey Newton's third law.

21.19. IDENTIFY: Apply Coulomb’s law to calculate the force each of the two charges exerts on the third charge. Add these forces as vectors.

SET UP: The three charges are placed as shown in Figure 21.19a.

EXECUTE: Like charges repel and unlike attract, so the free-body diagram for \( q_3 \) is as shown in Figure 21.19b.

\[ F_1 = \frac{1}{4\pi \varepsilon_0} \frac{|q_1 q_2|}{r_{13}^2} \]

\[ F_2 = \frac{1}{4\pi \varepsilon_0} \frac{|q_2 q_3|}{r_{23}^2} \]

Figure 21.19a

Figure 21.19b
\[
F_1 = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left( \frac{1.50 \times 10^{-9} \text{ C}}{0.200 \text{ m}} \right) = 1.685 \times 10^{-6} \text{ N}
\]

\[
F_2 = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left( \frac{3.20 \times 10^{-9} \text{ C}}{0.400 \text{ m}} \right) = 8.988 \times 10^{-7} \text{ N}
\]

The resultant force is \( \vec{R} = \vec{F}_1 + \vec{F}_2 \).

\( R_x = 0 \).

\[
R_y = F_1 + F_2 = 1.685 \times 10^{-6} \text{ N} + 8.988 \times 10^{-7} \text{ N} = 2.58 \times 10^{-6} \text{ N}.
\]

The resultant force has magnitude \( 2.58 \times 10^{-6} \text{ N} \) and is in the –y-direction.

**EVALUATE:** The force between \( q_1 \) and \( q_2 \) is attractive and the force between \( q_2 \) and \( q_3 \) is repulsive.

**21.25. IDENTIFY:** \( F = |q|E \). Since the field is uniform, the force and acceleration are constant and we can use a constant acceleration equation to find the final speed.

**SET UP:** A proton has charge \(+e\) and mass \( 1.67 \times 10^{-27} \text{ kg} \).

**EXECUTE:**

(a) \( F = (1.60 \times 10^{-19} \text{ C})(2.75 \times 10^3 \text{ N/C}) = 4.40 \times 10^{-16} \text{ N} \)

\[ a = \frac{F}{m} = \frac{4.40 \times 10^{-16} \text{ N}}{1.67 \times 10^{-27} \text{ kg}} = 2.63 \times 10^{11} \text{ m/s}^2 \]

(b) \( v_y = v_{0y} + at \) gives \( v = (2.63 \times 10^{11} \text{ m/s}^2)(1.00 \times 10^{-6} \text{ s}) = 2.63 \times 10^5 \text{ m/s} \)

**EVALUATE:** The acceleration is very large and the gravity force on the proton can be ignored.

**21.33. IDENTIFY:** Eq. (21.3) gives the force on the particle in terms of its charge and the electric field between the plates. The force is constant and produces a constant acceleration. The motion is similar to projectile motion; use constant acceleration equations for the horizontal and vertical components of the motion.

(a) **SET UP:** The motion is sketched in Figure 21.33a.

\[ \vec{F} = q\vec{E} \] and \( q \) negative gives that \( \vec{F} \) and \( \vec{E} \) are in opposite directions, so \( \vec{F} \) is upward. The free-body diagram for the electron is given in Figure 21.33b.

\[ a = \frac{eE}{m} \]

**EXECUTE:** \( \sum F_y = ma_y \)

\[
F = eE \\
\sum F_y = ma
\]

**Figure 21.33b**

Solve the kinematics to find the acceleration of the electron: Just misses upper plate says that \( x - x_0 = 2.00 \text{ cm} \) when \( y - y_0 = +0.500 \text{ cm} \).

**x-component**

\( v_{0x} = v_x = 1.60 \times 10^8 \text{ m/s} \), \( a_x = 0 \), \( x - x_0 = 0.0200 \text{ m}, t = ? \)

\[ t = \frac{x - x_0}{v_{0x}} = \frac{0.0200 \text{ m}}{1.60 \times 10^8 \text{ m/s}} = 1.25 \times 10^{-6} \text{ s} \]

In this same time \( t \) the electron travels 0.0050 m vertically:
\[ y - y_0 = v_y t + \frac{1}{2} a_y t^2 \]
\[ a_y = \frac{2(y - y_0)}{t^2} = \frac{2(0.0050 \text{ m})}{(1.25 \times 10^{-8} \text{ s})^2} = 6.40 \times 10^{13} \text{ m/s}^2 \]

(This analysis is very similar to that used in Chapter 3 for projectile motion, except that here the acceleration is upward rather than downward.) This acceleration must be produced by the electric-field force: \( eE = ma \)

\[ E = \frac{ma}{e} = \frac{(9.109 \times 10^{-31} \text{ kg})(6.40 \times 10^{13} \text{ m/s}^2)}{1.602 \times 10^{-19} \text{ C}} = 364 \text{ N/C} \]

Note that the acceleration produced by the electric field is much larger than \( g \), the acceleration produced by gravity, so it is perfectly ok to neglect the gravity force on the electron in this problem.

(b) \[ a = \frac{eE}{m_p} = \frac{(1.602 \times 10^{-19} \text{ C})(364 \text{ N/C})}{1.673 \times 10^{-5} \text{ kg}} = 3.49 \times 10^{19} \text{ m/s}^2 \]

This is much less than the acceleration of the electron in part (a) so the vertical deflection is less and the proton won’t hit the plates. The proton has the same initial speed, so the proton takes the same time \( t = 1.25 \times 10^{-8} \text{ s} \) to travel horizontally the length of the plates. The force on the proton is downward (in the same direction as \( \vec{E} \), since \( q \) is positive), so the acceleration is downward and \( a_y = -3.49 \times 10^{19} \text{ m/s}^2 \).

\[ y - y_0 = v_y t + \frac{1}{2} a_y t^2 = \frac{1}{2}(-3.49 \times 10^{19} \text{ m/s}^2)(1.25 \times 10^{-8} \text{ s})^2 = -2.73 \times 10^{-6} \text{ m} \]

The displacement is \( 2.73 \times 10^{-6} \text{ m} \), downward.

(c) Evaluate: The displacements are in opposite directions because the electron has negative charge and the proton has positive charge. The electron and proton have the same magnitude of charge, so the force the electric field exerts has the same magnitude for each charge. But the proton has a mass larger by a factor of 1836 so its acceleration and its vertical displacement are smaller by this factor.

21.40. Identify: The net force on each charge must be zero.

Set Up: The force diagram for the -6.50 \( \mu \text{C} \) charge is given in Figure 21.40. \( F_k \) is the force exerted on the charge by the uniform electric field. The charge is negative and the field is to the right, so the force exerted by the field is to the left. \( F_q \) is the force exerted by the other point charge. The two charges have opposite signs, so the force is attractive. Take the +x axis to be to the right, as shown in the figure.

Execute: (a) \[ F = \left| F \right| = (6.50 \times 10^{-6} \text{ C})(1.85 \times 10^3 \text{ N/C}) = 1.20 \times 10^1 \text{ N} \]

\[ F_q = k \frac{|q_1 q_2|}{r^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(6.50 \times 10^{-6} \text{ C})(8.75 \times 10^{-6} \text{ C})}{(0.0250 \text{ m})^2} = 8.18 \times 10^3 \text{ N} \]

\[ \sum F_x = 0 \text{ gives } T + F_q - F_k = 0 \text{ and } T = F_k - F_q = 382 \text{ N} \]

(b) Now \( F_q \) is to the left, since like charges repel.

\[ \sum F_x = 0 \text{ gives } T - F_q - F_k = 0 \text{ and } T = F_k + F_q = 2.02 \times 10^3 \text{ N} \]

Evaluate: The tension is much larger when both charges have the same sign, so the force one charge exerts on the other is repulsive.

Figure 21.40
21.50. **Identify:** Apply Eq.(21.7) to calculate the field due to each charge and then calculate the vector sum of those fields.

**Set up:** The fields due to \( q_1 \) and to \( q_2 \) are sketched in Figure 21.50.

**Execute:**

\[
\vec{E}_1 = \frac{1}{4\pi\varepsilon_0} \left( \frac{6.00 \times 10^{-9} \text{ C}}{(0.6 \text{ m})^2} \right) (-\hat{i}) = -150\hat{i} \text{ N/C}.
\]

\[
\vec{E}_2 = \frac{1}{4\pi\varepsilon_0} \left( \frac{4.00 \times 10^{-9} \text{ C}}{(1.0 \text{ m})^2} \right) \left( \frac{1}{(0.600) \hat{i}} + \frac{1}{(0.800) \hat{j}} \right) = (21.6\hat{i} + 28.8\hat{j}) \text{ N/C}.
\]

\[
\vec{E} = \vec{E}_1 + \vec{E}_2 = (-128.4 \text{ N/C})\hat{i} + (28.8 \text{ N/C})\hat{j}.
\]

\[
E = \sqrt{(128.4 \text{ N/C})^2 + (28.8 \text{ N/C})^2} = 131.6 \text{ N/C} \quad \text{at} \quad \theta = \tan^{-1} \left( \frac{28.8}{128.4} \right) = 12.6^\circ \text{ above the } -x \text{ axis and therefore } 196.2^\circ \text{ counterclockwise from the } +x \text{ axis.}
\]

**Evaluate:** \( \vec{E}_1 \) is directed toward \( q_1 \) because \( q_1 \) is negative and \( \vec{E}_2 \) is directed away from \( q_2 \) because \( q_2 \) is positive.

\[
\vec{E} = \vec{E}_1 + \vec{E}_2 = (-128.4 \text{ N/C})\hat{i} + (28.8 \text{ N/C})\hat{j} = 131.6 \text{ N/C} \quad \text{at} \quad \theta = 12.6^\circ \text{ above the } -x \text{ axis and therefore } 196.2^\circ \text{ counterclockwise from the } +x \text{ axis.}
\]

\[
\theta = \tan^{-1} \left( \frac{28.8}{128.4} \right) = 12.6^\circ \text{ above the } -x \text{ axis and therefore } 196.2^\circ \text{ counterclockwise from the } +x \text{ axis.}
\]

21.75. **Identify:** Use Coulomb's law for the force that one sphere exerts on the other and apply the 1st condition of equilibrium to one of the spheres.

**(a) Set up:** The placement of the spheres is sketched in Figure 21.75a.

The free-body diagrams for each sphere are given in Figure 21.75b.

\[
F_c \quad \text{is the repulsive Coulomb force exerted by one sphere on the other.}
\]
(b) EXECUTE: From either force diagram in part (a): \[ \sum F_y = ma_y \]

\[ T \cos 25.0^\circ - mg = 0 \quad \text{and} \quad T = \frac{mg}{\cos 25.0^\circ} \]

\[ \sum F_y = ma_y \quad \text{and} \quad T \sin 25.0^\circ - F_y = 0 \quad \text{and} \quad F_y = T \sin 25.0^\circ \]

Use the first equation to eliminate \( T \) in the second:

\[ F_y = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} \quad \text{and} \quad F_y = \frac{1}{4\pi \varepsilon_0} \frac{q^2}{(2(1.20 \text{ m}) \sin 25.0^\circ)^2} \]

Combine this with \( F_y = mg \tan 25.0^\circ \) and get

\[ q = (2.40 \text{ m}) \sin 25.0^\circ \sqrt{\frac{mg \tan 25.0^\circ}{(1/4\pi \varepsilon_0)}} \]

\[ q = (2.40 \text{ m}) \sin 25.0^\circ \sqrt{\frac{[15.0 \times 10^{-3} \text{ kg}] [9.80 \text{ m/s}^2] \tan 25.0^\circ}{8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}} = 2.80 \times 10^{-6} \text{ C} \]

(c) The separation between the two spheres is given by \( 2L \sin \theta \). \( q = 2.80 \mu \text{C} \) as found in part (b).

\[ F_y = (1/4\pi \varepsilon_0) \frac{q^2}{(2L \sin \theta)^2} \quad \text{and} \quad F_y = mg \tan \theta \].

\[ (\sin \theta)^2 \tan \theta = \frac{1}{4\pi \varepsilon_0} \frac{q^2}{4Lmg} \]

\[ \left(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \left(\frac{2.80 \times 10^{-6} \text{ C}}{4(0.600 \text{ m}) \left(15.0 \times 10^{-3} \text{ kg}\right)[9.80 \text{ m/s}^2]}\right) = 0.3328. \]

Solve this equation by trial and error. This will go quicker if we can make a good estimate of the value of \( \theta \) that solves the equation. For \( \theta \) small, \( \tan \theta \approx \sin \theta \). With this approximation the equation becomes \( \sin^2 \theta = 0.3328 \) and \( \sin \theta = 0.6930 \), so \( \theta = 43.9^\circ \). Now refine this guess:

\[ \begin{array}{c|c}
\theta & \sin^2 \theta \tan \theta \\
\hline
45.0^\circ & 0.5000 \\
40.0^\circ & 0.3467 \\
39.6^\circ & 0.3361 \\
39.5^\circ & 0.3335 \\
39.4^\circ & 0.3309 \\
\end{array} \]

so \( \theta = 39.5^\circ \)

EVALUATE: The expression in part (c) says \( \theta \to 0 \) as \( L \to \infty \) and \( \theta \to 90^\circ \) as \( L \to 0 \). When \( L \) is decreased from the value in part (a), \( \theta \) increases.

21.89. IDENTIFY: Divide the charge distribution into infinitesimal segments of length \( dx \). Calculate \( E_x \) and \( E_y \) due to a segment and integrate to find the total field.

SET UP: The charge \( dQ \) of a segment of length \( dx \) is \( dQ = (Q/a) dx \). The distance between a segment at \( x \) and the charge \( q \) is \( a + r - x \). \( (1 - y)^{-1} \approx 1 + y \) when \( |y| < 1 \).

EXECUTE: (a) \[ dE_x = \frac{1}{4\pi \varepsilon_0} \frac{dQ}{(a + r - x)^2} \quad \text{so} \quad E_x = \frac{1}{4\pi \varepsilon_0} \int \frac{Q dx}{a(a + r - x)^2} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{a} \left( \frac{1}{r - a} - \frac{1}{a + r} \right). \]

\[ a + r = x, \quad \text{so} \quad E_x = \frac{1}{4\pi \varepsilon_0} \frac{Q}{a} \left( \frac{1}{x - a} - \frac{1}{x} \right). \]

\[ E_y = 0. \]

(b) \[ \vec{F} = q \vec{E} = \frac{1}{4\pi \varepsilon_0} \frac{qQ}{a} \left( \frac{1}{x - a} - \frac{1}{x} \right) \hat{i}. \]
EVALUATE: \( (c) \) For \( x \gg a \),
\[
F = \frac{kQ}{ax} ((1 - a/x)^{-1} - 1) = \frac{kQ}{ax} (1 + a/x + \cdots - 1) \approx \frac{kQ}{x^2} \approx \frac{1}{4\pi\varepsilon_0} \frac{qQ}{r^2}.
\]
(Note that for \( x \gg a \), \( r = x - a \approx x \).) The charge distribution looks like a point charge from far away, so the force takes the form of the force between a pair of point charges.

21.96. IDENTIFY: Divide the semicircle into infinitesimal segments. Find the electric field \( dE \) due to each segment and integrate over the semicircle to find the total electric field.

SET UP: The electric fields along the \( x \)-direction from the left and right halves of the semicircle cancel. The remaining \( y \)-component points in the negative \( y \)-direction. The charge per unit length of the semicircle is
\[
\lambda = \frac{Q}{\pi a} \quad \text{and} \quad dE = \frac{k\lambda}{a^2} \frac{dl}{\sin \theta} = \frac{k\lambda}{a} d\theta.
\]

EXECUTE: \( dE_y = dE \sin \theta = \frac{k\lambda}{a} \sin \theta d\theta \). Therefore,
\[
E_y = \frac{2k\lambda}{a} \int_0^{\pi/2} \sin \theta d\theta = \frac{2k\lambda}{a} [-\cos \theta]_0^{\pi/2} = \frac{2k\lambda}{a} = \frac{2kQ}{\pi a^2}, \quad \text{in the \(-y\)-direction}.
\]

EVALUATE: For a full circle of charge the electric field at the center would be zero. For a quarter-circle of charge, in the first quadrant, the electric field at the center of curvature would have nonzero \( x \) and \( y \) components. The calculation for the semicircle is particularly simple, because all the charge is the same distance from point \( P \).