CHAPTER 21
ELECTRIC CHARGE AND
ELECTRIC FIELD
TWO BASIC CONCEPTS:

COULOMB’S LAW

\[ F = k \frac{Q_1 Q_2}{r^2} \]

ELECTRIC FIELD

\[ E = \frac{F}{q} \]
ATOMS – are neutral

(a) Neutral lithium atom (Li):
- 3 protons (3+)
- 4 neutrons
- 3 electrons (3−)

Electrons equal protons:
Zero net charge

Fig 21.4
IONS – are charged

- Protons (+)  - Neutrons
- Electrons (−)

(b) Positive lithium ion (Li⁺):
- 3 protons (3+)
- 4 neutrons
- 2 electrons (2−)

Fewer electrons than protons:
Positive net charge
SOME MATERIALS –

e’s move easily – conductors
metals

SOME MATERIALS –

e’s don’t move easily – insulators
glass
wood
FORCES ON CHARGED OBJECTS

LIKE CHARGES REPEL

UNLIKE CHARGES ATTRACT

(b) Interactions between point charges

\[ \vec{F}_{2 \text{ on } 1} = -\vec{F}_{1 \text{ on } 2} \]

\[ F_{1 \text{ on } 2} = F_{2 \text{ on } 1} = k \frac{|q_1 q_2|}{r^2} \]

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USE COULOMB’S LAW

\[ F = k \frac{Q_1 Q_2}{r^2} \]

Example 21.2

Two point charges \( q_1 = 25 \text{ nC} \) and \( q_2 = -75 \text{ nC} \) are separated by a distance of 3.0 cm. Find the magnitude and direction of the electric force that \( q_1 \) exerts on \( q_2 \).
(a) The two charges

\[ F = k \frac{Q_1 Q_2}{r^2} \]

\[ = (9 \times 10^9) \frac{(+25 \times 10^{-9})(-75 \times 10^{-9})}{(0.030m)^2} \]

\[ = 0.01875N \]

Attractive Force
Also

Newton’s third law – if body A exerts a force on body B, then body B exerts an equal and opposite force on body A.
ELECTRIC FIELD

Electric force per unit charge.

\[ E = \frac{F}{q} \]

Or

\[ E = \frac{F}{q_{\text{test}}} \]
In direction that positive charge would move.

For more than one charge the total electric field equals the vector sum of all electric fields due to each charge.
For two charges $q_1$ and $q_{\text{test}}$

\[ F = k \frac{q_1 q_{\text{test}}}{r^2} \]

So

\[ E = \frac{F}{q_{\text{test}}} = \frac{k \frac{q_1 q_{\text{test}}}{r^2}}{q_{\text{test}}} \]

Therefore
\[ E = k \frac{q_1}{r^2} \]

Or

\[ E = k \frac{q}{r^2} \]
Also

\[ k = \frac{1}{4\pi \varepsilon_0} \]

Where

\[ \varepsilon_0 = 8.854 \times 10^{-12} \frac{C^2}{Nm^2} \]

Therefore can write

\[ F = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} \]
And

\[ E = \frac{1}{4\pi \varepsilon_0 \, r^2} \frac{q}{r^2} \]
ELECTRIC FIELD LINES

1. IN DIRECTION OF FIELD (POSITIVE TEST CHARGE MOVES ALONG LINE.)

2. NUMBER OF LINES PROPORTIONAL TO ELECTRIC FIELD.

3. ELECTRIC FIELD LINES START ON POSITIVE CHARGE AND END ON NEGATIVE CHARGE.
(a) A single positive charge

Field lines always point away from (+) charges and toward (−) charges.

Insert fig 21.29
(b) Two equal and opposite charges (a dipole)

Field lines always point away from (+) charges and toward (−) charges.

At each point in space, the electric field vector is tangent to the field line passing through that point.
Example 21.2 (continued)

Find the electric field 3.0 cm from an electric charge
$q_1 = +25 \text{ nC.}$

$r = 3 \text{ cm}$
What is $E$ at distance $r$ due to $q_1$?

$$E = \frac{1}{4\pi \epsilon_0} \frac{q}{r^2} = \frac{1}{4\pi \epsilon_0} \frac{25 \times 10^{-9}}{0.03^2}$$

$$E = 2.5 \times 10^5 \frac{N}{C}$$

**DIRECTION?**

Away from positive $q_1$
If we place a charge -75nC at a point 3 cm away from $q_1$ what is the force on this charge?

(a) The two charges

$q_1$  \hspace{2cm} q_2$

$+$  \hspace{2cm} -$
The electric field at $q_2$ is 
$2.5 \times 10^5 \, \text{N/C}$

Therefore

$$F = qE$$

$$= (-75 \times 10^{-9} \, \text{C})(2.5 \times 10^5 \, \text{N/C})$$

$$= 0.01875 \, \text{N}$$

WE OBTAINED FORCE IN TWO WAYS
Example - Field from two charges

$Q_1 = -50\mu C$ at $x=0.52m, y=0$

$Q_2 = 50\mu C$ at $x=0, y=0$ and

find $E$ at point $A$ ($x=0, y=0.3m$).
\[ E_{A1} = \frac{k(-50 \times 10^{-6} \text{C})}{(0.6\text{m})^2} \]

\[ = -1.25 \times 10^6 \frac{\text{N}}{\text{C}} \]

(towards \(Q_1\))

\[ E_{A2} = \frac{k(50 \times 10^{-6} \text{C})}{(0.3\text{m})^2} \]

\[ = 5 \times 10^6 \frac{\text{N}}{\text{C}} \]

(away from \(Q_2\))
\[ E_{Ax} = E_{A1} \cos 30^0 \]
\[ = 1.1 \times 10^6 \, \text{N/C} \]
\[ E_{Ay} = E_{A2} - E_{A1} \sin 30^0 \]
\[ = 4.4 \times 10^6 \frac{N}{C} \]

\[ E_A = \sqrt{E_{Ax}^2 + E_{Ay}^2} \]
\[ = \sqrt{(1.1 \times 10^6)^2 + (4.4 \times 10^6)^2} \]

\[ E_A = 4.5 \times 10^6 \frac{N}{C} \]
\[ \tan \phi = \frac{E_{Ay}}{E_{Ax}} = \frac{4.4 \times 10^6 \text{N}/\text{C}}{1.1 \times 10^6 \text{N}/\text{C}} = 4 \]

\[ \phi = 76^0 \]
NOW CALCULATE THE FORCE ON A CHARGE $Q_3 = 50\mu\text{C}$ PLACED AT POINT A.

\[ F = qE = (50 \times 10^{-6} \text{C}) \times 4.5 \times 10^6 \frac{N}{\text{C}} \]
\[ = 225N \]

WHAT DIRECTION?
HAVE BEEN TALKING ABOUT POINT CHARGES.

WHAT ABOUT CONTINUOUS CHARGES?

\[ dE = k \frac{dQ}{r^2} \]

\[ dE = \frac{1}{4\pi \varepsilon_0} \frac{dQ}{r^2} \]
Consider a ring of uniform charge

![Diagram of a ring of uniform charge](image)

**Fig 21.24**

**BY SYMOMETRY** \( E_y = 0 \)

\[ dE_x = dE \cos \alpha \]
\[ dE = \frac{1}{4\pi \varepsilon_0} \frac{dQ}{r^2} \]

\[ dQ = \frac{Q}{2\pi a} \, ds = \lambda ds \]

Where \( \lambda = \frac{Q}{2\pi a} \)

\[ dE = \frac{1}{4\pi \varepsilon_0} \frac{\lambda ds}{r^2} \]

\[ E_x = \int dE_x = \int dE \cos \alpha \]
\[ E_x = \frac{1}{4\pi \varepsilon_0} \lambda \int \frac{ds}{r^2} \cos \alpha \]

\[ \cos \alpha = \frac{x}{r} \]

\[ E_x = \frac{1}{4\pi \varepsilon_0} \lambda \int \frac{ds \, x}{r^2 \, r} \]

\[ E_x = \frac{1}{4\pi \varepsilon_0} \frac{\lambda x}{r^3} \int_0^{2\pi a} ds \]

\[ E_x = \frac{1}{4\pi \varepsilon_0} \frac{\lambda x}{r^3} 2\pi a \]
Put in for \(\lambda = \frac{Q}{2\pi a}\)

\[
E_x = \frac{1}{4\pi \varepsilon_0} \frac{Qx}{r^3}
\]

\[
r = (x^2 + a^2)^{1/2}
\]

Finally

\[
E_x = \frac{1}{4\pi \varepsilon_0} \frac{Qx}{(x^2 + a^2)^{3/2}}
\]
THIS IS THE ELECTRIC FIELD AS A FUNCTION OF $x$ FOR A RING OF CHARGE WITH RADIUS $a$. 
START WITH THAT EQUATION AND FIND THE ELECTRIC FIELD ALONG THE AXIS OF A DISK WITH TOTAL CHARGE $Q$ AND RADIUS OF $R$.

INSERT FIG 21.26

ONCE AGAIN BY SYMETRY $E_y = 0$
Consider a small ring in the disk as shown.

For the ring:

Radius = r

Thickness = dr

Charge on ring = dQ

WE CAN WRITE FOR THE CONTRIBUTION TO THE ELECTRIC FIELD AT P IN THE x DIRECTION
\[ dE_x = \frac{1}{4\pi\varepsilon_0} \frac{dQ}{(x^2 + r^2)^{3/2}} \]

TO FIND THE TOTAL FIELD INTEGRATE
USE THE CHARGE DENSITY \( \sigma \)

THEN

\[ \sigma = \frac{Q}{\pi R^2} \]

CHARGE ON RING
\[ dQ = \sigma 2\pi r dr \]

\[ dE_x = \frac{1}{4\pi \varepsilon_0} \frac{\sigma 2\pi r dr \ x}{(x^2 + r^2)^{3/2}} \]

\[ dE_x = \frac{\sigma x}{2\varepsilon_0} \frac{r dr}{(x^2 + r^2)^{3/2}} \]

\[ E_x = \frac{\sigma x}{2\varepsilon_0} \int_0^R \frac{r dr}{(x^2 + r^2)^{3/2}} \]
\[ E_x = \frac{\sigma x}{2\varepsilon_0} \left[ -\frac{1}{(x^2 + r^2)^{1/2}} \right]^R_0 \]

\[ E_x = \frac{\sigma x}{2\varepsilon_0} \left[ -\frac{1}{(x^2 + R^2)^{1/2}} - \left( -\frac{1}{x} \right) \right] \]

\[ E_x = \frac{\sigma x}{2\varepsilon_0} \left[ -\frac{1}{x \left( 1 + \frac{R^2}{x^2} \right)^{1/2}} - \left( -\frac{1}{x} \right) \right] \]
\[ E_x = \frac{\sigma}{2\epsilon_0} \left[ -\frac{1}{\left(1 + \frac{R^2}{x^2}\right)^{1/2}} - (-1) \right] \]

\[ E_x = \frac{\sigma}{2\epsilon_0} \left[ 1 - \frac{1}{\left(1 + \frac{R^2}{x^2}\right)^{1/2}} \right] \]
NOW CONSIDER AN INFINITE SHEET WITH CHARGE DENSITY $\sigma$

WHAT IS THE ELECTRIC FIELD A DISTANCE $x$ FROM THE SHEET?

FOR A DISK OF RADIUS $R$

$$E_x = \frac{\sigma}{2\epsilon_0} \left[ 1 - \frac{1}{\left(1 + \frac{R^2}{x^2}\right)^{1/2}} \right]$$
LET RADIUS EXPAND TO A VALUE OF INFINITY

\[ R \rightarrow \infty \]

\[ E_x = \frac{\sigma}{2\varepsilon_0} \left[ 1 - \frac{1}{(1 + \frac{\infty}{x^2})^{1/2}} \right] = \frac{\sigma}{2\varepsilon_0} \]
ANYWHERE NEAR AN INFINITE SHEET OF CHARGE THE ELECTRIC FIELD WILL BE

\[ \frac{\sigma}{2\varepsilon_0} \]