We have derived the addition of velocity equation for motion parallel to the motion of the moving frame

\[ u'_x = \frac{u_x - v}{1 - \frac{vu_x}{c^2}} \]

Now we need the equation for motion perpendicular to the direction of motion of the moving frame. It is

\[ u'_y = \frac{u_y}{\gamma \left( 1 - \frac{vu_x}{c^2} \right)} \]
Consider two observers each with a ball. The balls are identical. The two observers are moving with respect to each other. For example:

Looking down on the railroad tracks:
There are two identical balls. One is with the observer on the railroad car (moving frame) and one is with the observer on the ground.

The balls are thrown perpendicular to the tracks in a way such that they will collide as shown.

From the view of the observer on the ground the equation for conservation of momentum is

\[ 2(mu_y) = 2(mu_y') \]
Momentum is not conserved if $u_y \neq u'_y$

But $u'_y$ is

$$u'_y = \frac{u_y}{\gamma \left(1 - \frac{vu_x}{c^2}\right)}$$

$$u_x = 0$$

Thus

$$u'_y = \frac{u_y}{\gamma}$$

Not

$$u_y = u'_y$$
But momentum is conserved if

\[ m_{\text{moving}} = \gamma m_{\text{rest}} \]

This states that a moving mass has a larger mass than the same object when not moving.

The conservation of momentum holds if we

Have a different definition for mass

\[ m_{\text{moving}} = \gamma m_{\text{rest}} \]
The equations you used in Physics 201 come from considering mass as a constant. But if mass is dependent on velocity all changes.

In Physics 201

\[ \text{Force} = ma \]

\[ \text{Kinetic Energy} = \frac{1}{2} m v^2 \]

Relativity

\[ E = mc^2 \]