CHAPTER 29
ATOMIC STRUCTURE
QUANTUM NUMBERS

We have assumed circular orbits

Then for hydrogen using the principle quantum number \( n \)

The Energy is

\[
E_n = -\frac{1}{n^2} \ 13.6 \ eV
\]

The Angular Momentum is

\[
L_n = n \frac{h}{2\pi}
\]
If you know $n$ you know both energy and angular momentum
Only one quantum number.

Sommerfeld in 1915 considered elliptical orbits

For hydrogen atoms with $n$ all same energy.

But angular momentum will be different – maximum for circular orbit
minimum for straight line orbit (zero)

Need another quantum number

Have principal quantum number

$$n = 1, 2, 3, 4, 5, \ldots \ldots \ldots \ldots \ldots \ldots$$
Now we introduce angular momentum quantum number

\[ l = 0, 1, 2, 3, \ldots \ldots (n - 1) \]

This second quantum number is for angular momentum.

The third quantum number is needed because the electron in a closed orbit is a current loop. A current loop gives a magnetic moment. The Stern-Gerlach Experiment showed magnetic moments only in specific directions.

Introduce the third quantum number

magnetic quantum number
\[ m_l = -l, \ldots \ldots \ldots -1, 0, 1, \ldots \ldots + l \]

The electron in orbit has a magnetic moment. For this another quantum number is needed.

Spin quantum number

\[ m_s = -\frac{1}{2} \text{ or } +\frac{1}{2} \]

With the four quantum numbers you can describe the complete state of the atom.
A system is used to identify the state of the atom.

For an electron in an atom with $l = 0$ is said to be in an $s$ state.

For an electron in an atom with $l = 1$ is said to be in an $p$ state.

For an electron in an atom with $l = 2$ is said to be in an $d$ state.

For an electron in an atom with $l = 3$ is said to be in an $e$ state.

Thus for

\[ l = 0, 1, 2, 3, 4, 5 \]

state is $s$ $p$ $d$ $e$ $f$ $g$
PAULI EXCLUSION PRINCIPLE

For Hydrogen – only one electron.

Other elements have more electrons

PAULI proposed rule that explains chemical behavior of elements.

IN ANY ATOM NO TWO ELECTRONS CAN HAVE THE SAME SET OF QUANTUM NUMBERS.
The quantum numbers and their relationship needed to explain the experiments are:

\[ n = 1, 2, 3, 4, 5, \ldots \ldots \ldots \ldots \ldots \]

\[ l = 0, 1, 2, 3, \ldots (n - 1) \]

\[ m_l = -l, \ldots -1, 0, 1, \ldots \ldots + l \]

\[ m_s = -\frac{1}{2} \text{ or } +\frac{1}{2} \]

These relationships and the Pauli Principle will give:

Hydrogen (1 electron)

Electron state

\[ n = 1, l = 0, m_l = 0, m_s = \frac{1}{2} \]
Helium (2 electrons)

1st electron

\[ n = 1, l = 0, m_l = 0, m_s = \frac{1}{2} \]

2nd electron

\[ n = 1, l = 0, m_l = 0, m_s = -\frac{1}{2} \]
Lithium (3 electrons)

1st electron

\[ n = 1, l = 0, m_l = 0, m_s = \frac{1}{2} \]

2nd electron

\[ n = 1, l = 0, m_l = 0, m_s = \frac{1}{2} \]

3rd electron

\[ n = 1, l = 0, m_l = 0, m_s = \frac{1}{2} \]
Beryllium (4 electrons)

1st electron
\[ n = 1, l = 0, m_l = 0, m_s = \frac{1}{2} \]

2nd electron
\[ n = 1, l = 0, m_l = 0, m_s = -\frac{1}{2} \]

3rd electron
\[ n = 2, l = 0, m_l = 0, m_s = \frac{1}{2} \]

4th electron
\[ n = 2, l = 0, m_l = 0, m_s = -\frac{1}{2} \]

detc.
Insert Table 29.1 in Thornton and Rex

Insert Table 29.2 in Thornton and Rex
From Quantum Mechanics it is found that the angular momentum vector can have only specific magnitudes.

These are given by:

\[ L = \sqrt{l(l + 1)} \frac{h}{2\pi} \]

And the components of the vector \( L \) (projections on an axis, for example \( z \)) can only be

\[ L_z = m_l \frac{h}{2\pi} \]
The conclusion must be that the angular momentum vector can only point in specific directions relative to some axis. (The axis could be defined by a magnetic field.)

The directions are shown in Figure 29.2

Insert Fig 29.2 from Young.