CHAPTER 26
INTERFERENCE AND DIFFRACTION

INTERFERENCE

CONSTRUCTIVE
DESTRUCTIVE

YOUNG’S EXPERIMENT

THIN FILMS

NEWTON’S RINGS

DIFFRACTION

SINGLE SLIT
MULTIPLE SLITS

RESOLVING POWER
IN PHASE                  180° OUT OF PHASE
CONSTRUCTIVE INTERFERENCE

Waves interfere constructively if their path lengths differ by an integral number of wavelengths:

\[ r_2 - r_1 = m\lambda \]

(b) Condition for constructive interference
DESTRUCTIVE INTERFERENCE

Waves interfere destructively if their path lengths differ by a half-integral number of wavelengths:

\[ r_2 - r_1 = (m + \frac{1}{2})\lambda. \]

(c) Condition for destructive interference
When two waves arrive at a point where path differences from the sources differ by one wavelength, two wavelengths, three wavelengths, etc. there will be constructive interference.

Or mathematically:

\[ r_2 - r_1 = m\lambda \text{ where } m = 0, \pm 1, \pm 2, \pm 3, \ldots \]
When two waves arrive at a point where path differences from the sources differ by \( \frac{1}{2} \) wavelength, \( \frac{3}{2} \) wavelength, \( \frac{5}{2} \) wavelength, etc. (Or half integer number of wavelengths) there will be destructive interference.

Or mathematically:

\[
r_2 - r_1 = \left( m + \frac{1}{2} \right) \lambda
\]

Where \( m = 0, \pm 1, \pm 2, \pm 3, \ldots \)
Many ways to produce two rays of coherent light.

(a) Interference of light waves passing through two slits
Consider path differences

In real situations, the distance $R$ to the screen is usually very much larger than the distance $d$ between the slits . . .

(b) Actual geometry (seen from the side)

Path difference $= d \sin \theta$
Constructive Interference

\[ d \sin \theta = m\lambda \]

where \( m = 0, \pm 1, \pm 2, \pm 3, \ldots \)

Destructive Interference

\[ d \sin \theta = \left( m + \frac{1}{2} \right) \lambda \]

where \( m = 0, \pm 1, \pm 2, \pm 3, \ldots \)
We can obtain the distance $y$ a bright line is below (or above) the center of the screen.

Use

**In real situations, the distance $R$ to the screen is usually very much larger than the distance $d$ between the slits . . .**

(b) Actual geometry (seen from the side)
If $R \gg d$

$$\tan \theta \approx \frac{y}{R}$$

Therefore

$$y = R \tan \theta$$

And for the $m^{th}$ bright line

$$y_m \approx R \tan \theta_m$$

For small angles

$$\tan \theta \approx \sin \theta$$

Therefore

$$y_m \approx R \sin \theta_m$$
Use this with

\[ d \sin \theta = m \lambda \]

Or

\[ \sin \theta_m = \frac{m \lambda}{d} \]

To get

\[ y_m \approx R \frac{m \lambda}{d} \]

where \( m = 0, \pm 1, \pm 2, \pm 3, \ldots \)

This is the equation that Young used to measure the wave length of light.
As was said many ways of producing two rays of coherent light. We have been discussing

(a) Interference of light waves passing through two slits

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Can also get two coherent rays from thin films.

Light reflected from the upper and lower surfaces of the film comes together in the eye at $P$ and undergoes interference.

Some colors interfere constructively and others destructively, creating the iridescent color bands we see.

(a) Interference between rays reflected from the two surfaces of a thin film
(b) The rainbow fringes of an oil slick on water
We will discuss the interference between rays coming from the two surfaces on opposite sides of a space filled with liquid (thin film) or air.

First: we need to know that when light reflects from a surface sometimes it undergoes a phase shift of $180^0$.

For example if a ray is in air ($n = 1.00$) and reflects at the surface with water ($n = 1.33$) back into the air it **does not** undergo a phase shift.

But if a ray is in glass ($n = 1.33$) and reflects at the surface with air ($n = 1.00$) back into the glass it undergoes a $180^0$ phase shift.
Rule

If ray goes from low $n$ material and reflects at surface of high $n$ material **no phase shift**.

If ray goes from high $n$ material and reflects at surface with low $n$ material **180 degree phase shift**.

\[
\text{If } n_b \xrightarrow{\text{reflected from}} n_a \text{ and } n_a > n_b
\]

No phase shift.

But

\[
\text{If } n_a \xrightarrow{\text{reflected from}} n_b \text{ and } n_a > n_b
\]

$180^\circ$ (half wavelength) phase shift
Now consider we have a film as shown

Assume perpendicular.

Film thickness where shown \( t \).
No phase shift at either surface or phase shift at both surfaces

Constructive interference if

$$2t = m\lambda \quad m = 0, 1, 2, 3, \ldots$$

But if one surface has phase shift and other does not

Destructive interference.

Or

No phase shift at either surface or phase shift at both surfaces

Destructive interference if

$$2t = \left( m + \frac{1}{2} \right) \lambda \quad m = 0, 1, 2, 3, \ldots$$
Example 26.3

\[ \lambda_0 = 500 \text{ nm} \]

Find separation of interference fringes.

One reflection (top) – no phase shift other (bottom) – phase shift.
Equation for destructive interference

\[ 2t = m\lambda \quad m = 0, 1, 2, 3, \ldots \]

Where glass touches – destructive interference.

From figure

\[ \frac{t}{x} = \frac{h}{l} \]

So

\[ t = \frac{xh}{l} \]

Therefore

\[ 2 \left( \frac{xh}{l} \right) = m\lambda \]

\[ x = m \left( \frac{l}{2h} \right) \lambda = m \frac{l\lambda}{2h} \]
\[ x = m \left[ \frac{(0.1m)(500 \times 10^{-9} m)}{2(0.02 \times 10^{-3} m)} \right] \]

\[ x = m \ (1.25 \ mm) \]
EXAMPLES OF INTERFERENCE PHENOMENA

Newton’s Rings

(a) A convex lens in contact with a glass plane
(b) Newton’s rings: circular interference fringes
Destructive interference occurs when
• the film is about $\frac{1}{4}\lambda$ thick and
• the light undergoes a phase change at both reflecting surfaces,
so that the two reflected waves emerge from the film about $\frac{1}{2}$ cycle out of phase.

$n_{\text{glass}} > n_{\text{film}} > n_{\text{air}}$

“Nonreflecting” film

Glass

Film

Air

$t = \frac{1}{4}\lambda$
DIFFRACTION

Light goes in straight line.

Therefore would expect sharp shadow of object on a screen.

But
Consider light going through single slit.

This shows a ray from the top of the slit and one from half way down the slit (middle).

For the two strips shown, the path difference to $P$ is $(a/2) \sin \theta$. When $(a/2) \sin \theta = 1/2$, the light cancels at $P$. This is true for the whole slit, so $P$ represents a dark fringe.

Or looking only at the slit

$\theta$ is usually very small, so we can use the approximations $\sin \theta = \theta$ and $\tan \theta = \theta$. Then the condition for a dark band is

$$y_m = R \frac{m\lambda}{a}.$$
The difference in path distance to the screen is

\[ \text{Path Diff.} = \frac{a}{2} \sin \theta \]

If the path difference is one half wave length – destructive interference.

Therefore the requirement for a dark fringe is

\[ \frac{a}{2} \sin \theta = \pm \frac{\lambda}{2} \]

\[ \sin \theta = \pm \frac{\lambda}{a} \]
If you divide the slit into 4 regions

If the path difference is one half wave length – destructive interference.

Therefore the requirement for a dark fringe is

\[
\frac{a}{4} \sin \theta = \pm \frac{\lambda}{2}
\]

\[
\sin \theta = \pm 2 \frac{\lambda}{a}
\]
Divide the slit into more sections and can show

The requirements for dark fringes are

\[
\sin \theta = m \frac{\lambda}{a} \quad (m = \pm 1, \pm 2, \pm 3, \ldots)
\]
If the wavelength of light is much smaller than the slit width (and normally it is) then

$$\frac{m\lambda}{a} \ll 1 \ (\text{very small})$$

Therefore

$$\theta \ll 1 \ (\text{very small})$$

And

$$\sin \theta \approx \theta$$

Therefore for dark fringes

$$\theta \approx \frac{m\lambda}{a} \ (m = \pm 1, \pm 2, \pm 3, \ldots)$$
Also for

For the two strips shown, the path difference to \( P \) is \((a/2) \sin \theta\).
When \((a/2) \sin \theta = 1/2\), the light cancels at \( P \). This is true for the whole slit, so \( P \) represents a dark fringe.

Use for distance to screen \( R = x \)

Then \( \tan \theta = \frac{y}{R} \)

And for the \( m \)th dark band

\[
\tan \theta_m = \frac{y_m}{R}
\]
If the screen is a distance from the slit such that

\[ R \gg y_m \]

And it normally is

Then \( \theta \ll 1 \) \text{ (very small)}

And \( \tan \theta_m \approx \theta_m \)

Therefore

\[ \theta_m \approx \frac{y_m}{R} \]
Then using

\[ \theta \approx \frac{ml}{a} \quad (m = \pm 1, \pm 2, \pm 3, \ldots) \]

\[ \theta_m \approx \frac{y_m}{R} \approx \frac{m\lambda}{a} \quad (m = \pm 1, \pm 2, \pm 3, \ldots) \]

Solve for \( y_m \)

\[ y_m \approx R \frac{m\lambda}{a} \quad (m = \pm 1, \pm 3, \pm 5, \ldots) \]

This is equation 26.10 in the book.
Multiple Slits

Maxima occur where the path difference for adjacent slits is a whole number of wavelengths: \( d \sin \theta = m\lambda \).

Path difference will be \( d \sin \theta \)
Constructive interference occurs for

\[ d \sin \theta = m\lambda \quad (m = 0, \pm 1, \pm 2, \pm 3, \ldots) \]

More complicated than two slit diffraction

Two slits produce one minimum between adjacent maxima.

(a) Two slits
Eight Slit

Eight slits produce larger, narrower maxima in the same locations, separated by seven minima.

(b) Eight slits
More – 16 slits

With sixteen slits, the maxima are still taller and narrower, with more intervening minima.

(c) Sixteen slits
DIFFRACTION GRATINGS

Prisms refract light different amounts depending on the wavelength of the light.

Note short wavelengths (blue) are refracted more than he long wavelengths (red).
Diffraction Gratings will do much the same.
The equation for the maxima is the same as for multiple slits.

\[ d \sin \theta = m\lambda \quad (m = 0, \pm 1, \pm 2, \pm 3, \ldots) \]

Note

\[ \sin \theta = \frac{m\lambda}{d} \]

Thus long wavelengths diffracted more than short wavelengths. (Opposite to prism.)
X-RAY DIFFRACTION

First experiments in 1912.

Some x rays are scattered as they pass through the crystal, forming an interference pattern on the film. (Most of the x rays pass straight through the crystal.)

(a) Basic setup for x-ray diffraction

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Crystal is uniform arrangement of atoms forming planes for diffraction.
Gives pattern

(b) Laue diffraction pattern for a thin section of quartz crystal
Consider the atoms as all lined up on the sites and represented by the dots shown.

(a) Scattering of waves from a rectangular array
Look at top row for example

Path lengths are equal when $\theta_a = \theta_r$

Interference from adjacent atoms in a row is constructive when the path lengths $a \cos \theta_a$ and $a \cos \theta_r$ are equal.

(b) Scattering from adjacent atoms in row
Therefore when the two angles are equal get constructive interference from diffraction from rows.

Then considering two rows one beneath the other.

Interference from atoms in adjacent rows is constructive when the path difference $2d \sin \theta$ is an integral number of wavelengths.

(c) Scattering from atoms in adjacent rows
Additional constructive interference when path lengths are equal to integer times wavelength of x-rays.

\[ 2dsin\theta = m\lambda \quad (m = 1, 2, 3, \ldots) \]

When both conditions are satisfied

1. \[ \theta_a = \theta_r \]

And

2. \[ 2dsin\theta = m\lambda \]

When these conditions are not satisfied radiation interferes destructively.

Therefore obtain diffraction pattern characteristic of crystal.
Scattering and thus diffraction can occur from various planes within the crystal.

Spacing of planes is $d = \frac{a}{\sqrt{2}}$.

Spacing of planes is $d = \frac{a}{\sqrt{3}}$. 

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CIRCULAR APERTURES

Light travelling through a small circular aperture is diffracted also.

$\theta_1$ is the angle between the center of the pattern and the first minimum.
The angular locations of the dark rings are given by:

\[ \sin \theta_1 = 1.22 \frac{\lambda}{D} \]

\[ \sin \theta_2 = 2.23 \frac{\lambda}{D} \]

\[ \sin \theta_3 = 3.24 \frac{\lambda}{D} \]

OBSERVING OBJECTS THROUGH VARIOUS SIZE APERTURES.
If the aperture in (a) were smaller 3 and 4 would appear as one object.

The larger the aperture the better the resolving power.

Lord Rayleigh proposed a criterion for telescopes (and other optical instruments) to give a measure for their capability.

Two objects can only be resolved if the center of the second just falls at a distance of the center of the first dark ring of the first.

The limit of resolution, $\theta_{res}$ for a telescope (or other optical instrument):

$$\theta_{res} = 1.22 \frac{\lambda}{D}$$