CHAPTER 21
ELECTROMAGNETIC
INDUCTION

BASIC CONCEPTS

Faraday’s Law

Lenz’s Law

Motional $emf$
Electric Charge Produces an

**Electric Field**

Moving Electric Charge Produces a

**Magnetic Field**

Now Changing Magnetic Field Produces an

**Emf**
a potential difference (voltage)

What is changing and causing the \textit{emf} is the magnetic flux.
We defined Electric Flux

$$\phi_{electric} = EA\cos \phi$$

Magnetic Flux is similar

$$\phi_{magnetic} = BA\cos \phi$$

Flux through an area
We will define a vector to represent area. The direction of the vector will be perpendicular to the surface. The length of the vector will be the area of the surface. The vector will be \( \mathbf{A} \).

The flux will be
First, flat area perpendicular to $B$

$$\phi_B = \sum B\Delta A \cos \theta = BA$$
For a surface parallel to $B$

$$
\phi_B = \sum B\Delta A\cos 90 = 0
$$

And for in between

$$
\phi_B = \sum B\Delta A\cos \phi
$$

$$
\phi_B = BA\cos \phi
$$
If this flux changes an *emf* will be induced in the area.

Consider a copper coil with one loop
\[ \phi_B = B\pi r^2 \]

For N loops

\[ \phi_B = NB\pi r^2 \]
FARADAY’S LAW

The emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit.

Or

\[ \varepsilon = \frac{\Delta \phi_B}{\Delta t} \]
If $\phi_B$ changes with time an emf, $\varepsilon$, will be produced.

Coil in magnetic field.
Flux through coil

\[ \phi_B = BA \cos \phi \]

Flux can change:

1. Magnitude of \( B \) changes.

2. Area of loop changes.
3. The angle $\phi$ changes.

4. Any combination of the three.
Example:
Consider a coil with 200 turns on a rectangular frame, 20 cm by 30 cm. The resistance of the coil is 2 Ω.
If \( B \) increases uniformly from zero to 0.5 \( \text{Wb}/m^2 \) in 0.8 s what is the current in the coil?

\[
A = (0.2m)(0.3m) = 0.060m^2
\]

At \( t = 0 \)

\( \phi_B = 0 \)

At \( t = 0.8s \) \( \phi_B = BA = \)

\[
(0.5\text{Wb}/m^2)0.060m^2 = 0.03\text{Wb}
\]
\[ |\varepsilon| = \frac{N\phi_B(t = 0) - N\phi_B(t = 0.8s)}{0.8s} \]

\[ |\varepsilon| = \frac{200(0.03\, \text{Wb})}{0.8s} \]

\[ |\varepsilon| = 7.5\, \text{V} \]

The current will be

\[ I = \frac{\varepsilon}{R} = \frac{7.5\, \text{V}}{2\, \Omega} = 3.75\, \text{A} \]
Lenz’s Law

A changing magnetic flux will cause an emf and if the emf is in a conductor there will be a current.

What will be the direction of the current?

Lenz’s Law will answer that question.
The polarity of the induced \emf is such that it tends to produce a current that will create a magnetic flux to oppose the change in magnetic flux through the loop.
a. Magnet moves toward loop. Magnet field is down and increasing. Induced \textit{emf} and thus current will be \textit{ccw} to \textbf{oppose} the increase.

The induced magnetic field is \textit{upward} to oppose the flux change. To produce this induced field, the induced current must be \textit{counterclockwise} as seen from above the loop.
b. Magnet moves away from loop. Magnetic field is up and but decreasing.

Induced current gives magnetic field to **oppose** the decrease.

The induced magnetic field is *downward* to oppose the flux change. To produce this induced field, the induced current must be **clockwise** as seen from above the loop.
Motional \textit{emf}

(a) Isolated moving rod

Charges in the moving rod are acted upon by a magnetic force $\vec{F}_B$...

... and the resulting charge separation creates a canceling electric force $\vec{F}_E$.

$F_B = quB$

$F_E = qE$

$\Delta x$
In time $\Delta t$ the rod will move $\Delta x$.

The area covered in $\Delta t$ will be

$$\Delta A = L\Delta x$$

$$\Delta x = v\Delta t$$

$$\Delta A = Lv\Delta t$$

Faraday’s Law
\[ \varepsilon = \frac{\Delta \phi_B}{\Delta t} \]

\[ \Delta \phi_B = B \Delta A = BLv\Delta t \]

\[ \varepsilon = \frac{BLv\Delta t}{\Delta t} = BLv \]

A conducting rod moving in a magnetic field will have an \textit{emf} Induced across it given by this equation.
Example: My airplane

Luscombe 8E
Built 1947
Wingspan 22 feet = 6.7m
Speed 100 mph = 44.7 m/s
If I am flying it at a point on earth where the vertical magnetic field is $5 \times 10^{-5}T$ what is the voltage drop across the wings?

$$\varepsilon = Blv$$

$$\varepsilon = (5 \times 10^{-5}T)(6.7m)(44.7m/s)$$

$$\varepsilon = 1.5 \times 10^{-2}V$$
Consider a loop around a long solenoid.
The current in the solenoid is increasing so the flux through the loop is increasing.

An \textit{emf} is induced in the loop by
INDUCTANCE

BASIC CONCEPTS

Mutual Inductance

Self Inductance

Magnetic Field Energy

Circuits with Inductors
Mutual Inductance

If \( I_1 \) changes with time \( t \)

\[
\frac{\Delta I_1}{\Delta t} \quad \text{yields} \quad \frac{\Delta B_1}{\Delta t} \quad \text{at } P
\]
Put second coil at $P$

\[
\frac{\Delta B_1}{\Delta t} \quad \text{yields} \quad \varepsilon_2 \quad \text{in coil \#2}
\]
\[ \varepsilon_2 \rightarrow I_2 \]

If \( I_1 \) changes with time

For example \( I_1 = I_0 \sin \omega t \)

Then \( B_1 = B_0 \sin \omega t \)

And

\[ \varepsilon_2 = -\frac{\Delta \Phi_B}{\Delta t} \]
Or \( \varepsilon_2 = -AB_0 \omega \cos \omega t \)

And

\[
I_2 = \frac{\varepsilon_2}{R_2} = \frac{-AB_0 \omega}{R_2} \cos \omega t
\]
\[ M = \frac{N_2 \Phi_{21}}{I_1} = \frac{N_2 \mu_0 \frac{N_1}{l} I_1 A}{I_1} \]

\[ M = N_2 \mu_0 \frac{N_1}{l} A = \frac{\mu_0 N_1 N_2 A}{l} \]

Only depends on geometry, number of turns, area and length.
Self Inductance

**Self-inductance:** If the current \( i \) in the coil is changing, the changing flux through the coil induces an emf in the coil.

If the current is changing then there will be a changing magnetic flux through the coil.
A changing magnetic flux will induce an *emf*.

\[ \varepsilon = -\frac{d\Phi}{dt} \]

Therefore we define in analogy to the mutual inductance

\[ L = \frac{N\Phi_B}{i} \]

This is Self-Inductance

Solve for \( \Phi_B \)
\[ \Phi_B = \frac{Li}{N} \]

Faraday’s Law

\[ \varepsilon = -N \frac{\Delta \Phi_B}{\Delta t} = -L \frac{\Delta i}{\Delta t} \]

Self-Induced emf

\[ \varepsilon = -L \frac{\Delta i}{\Delta t} \]
What is the inductance, \( L \), of the black coil?

\[
L = \frac{N \Phi_B}{l}
\]

\[
B_{\text{Solenoid}} = \mu_0 \frac{N_1}{l} I_1
\]

\[
\Phi_B = \mu_0 \frac{N_1}{l} I_1 A
\]

\[
L = \frac{N_1 \mu_0 \frac{N_1}{l} I_1 A}{I_1} = \mu_0 \frac{N_1^2}{l} A
\]
Consider the coil

(c) Inductor with increasing current $i$ flowing from $a$ to $b$: potential drops from $a$ to $b$.

\[ i \text{ increasing: } \frac{di}{dt} > 0 \]

\[ V_{ab} = L \frac{di}{dt} > 0 \]

Current starts at zero and increases to $I$. 
\[ \text{Power} = I^2 R = \frac{V^2}{R} = IV \]

Or

\[ P = I \varepsilon \]

And

\[ \varepsilon = -L \frac{\Delta I}{\Delta t} \]

\[ |P| = I \left( L \frac{\Delta I}{\Delta t} \right) = IL \frac{\Delta I}{\Delta t} \]

Power is work per unit time.

\[ P = \frac{\Delta W}{\Delta t} \]
\[ \Delta W = P \Delta t = IL \frac{\Delta I}{\Delta t} \Delta t = LI \Delta I \]

How much work to bring current through inductor from zero to \( I \)?

To calculate the work integrate from zero current to current \( I \)

We won’t do it but you may see how on the next page.
$$W = \int_{l=0}^{l=I} LI\,dl = L \int_{0}^{I} ld\ell$$

$$W = \frac{1}{2} LI^2$$

Work \quad \text{Goes to} \quad \text{Stored Energy}
The energy stored in the magnetic field is

$$U = \frac{1}{2} LI^2$$

Apply to solenoid
Volume = Al

\[ B = \mu_0 \frac{N_1}{l} I \]

So

\[ I = \frac{l}{\mu_0 N_1} B \]

\[ L = \mu_0 \frac{N_1^2 A}{l} \]

Put into equation for \( U \)

\[ U = \frac{1}{2} LI^2 = \frac{1}{2} \mu_0 \frac{N_1^2 A}{l} \left( \frac{l}{\mu_0 N_1} B \right)^2 \]
\[ U = \frac{1}{2} \frac{A l B^2}{\mu_0} \]

\[ U = \frac{1}{2} \frac{B^2}{\mu_0} (Al) \]

And

\[ Volume = Al \]

So

\[ \frac{Energy}{Volume} = \frac{1}{2} \frac{B^2}{\mu_0} (Al) \]

\[ u = \frac{1}{2} \frac{B^2}{\mu_0} \]

Energy stored in magnetic field \( B \) of coil.
R-L Circuit

Consider this circuit

Closing switch $S_1$ connects the $R-L$ combination in series with a source of emf $\mathcal{E}$.

Closing switch $S_2$ while opening switch $S_1$ disconnnects the combination from the source.
First consider that switch $S_1$ is closed and calculate how the current will increase with time.

Second, then while there is a current in the circuit consider opening $S_1$ and closing $S_2$ and calculate how the current will decay.

Close $S_1$
Switch $S_1$ is closed at $t = 0$.

Kirchhoff’s loop starting at the switch and going ccw.

At time $t = 0$ close switch.
\[ \varepsilon - iR - L \frac{\Delta i}{\Delta t} = 0 \]

\[ \frac{\Delta i}{\Delta t} = \frac{\varepsilon - iR}{L} \]

\[ \frac{\Delta i}{\Delta t} = \frac{\varepsilon}{L} - \frac{R}{L} i \]

At \( t = 0 \) \( i=0 \)

Then

\[ \left( \frac{\Delta I}{\Delta t} \right)_{t=0} = \frac{\varepsilon}{L} \]

After long time

\[ \frac{\Delta I}{\Delta t} = 0 \]
And

\[ I = \frac{\varepsilon}{R} \]
Switch $S_1$ is closed at $t = 0$.

\[
I = \frac{\mathcal{E}}{R}
\]

\[
I \left(1 - \frac{1}{e}\right)
\]

\[
t = \tau = \frac{L}{R}
\]
The L-C Circuit

The capacitor has been charged to an initial charge \( q \) and then placed across the inductor.
The L-C Circuit

Remember energy in

Capacitor \[ U_E = \frac{1}{2} \frac{q^2}{C} \]

And

Inductor \[ U_B = \frac{1}{2} Li^2 \]

Then study
Transformers

The induced emf *per turn* is the same in both coils, so we adjust the ratio of terminal voltages by adjusting the ratio of turns:

\[
\frac{V_2}{V_1} = \frac{N_2}{N_1}
\]