CHAPTER 20
MAGNETIC FIELD AND MAGNETIC FORCES

MAJOR CONCEPTS

MAGNETIC FIELD
MAGNETIC FORCE ON MOVING CHARGE
MAGNETIC FIELD LINES
MAGNETIC FLUX
MAGNETIC FORCE ON WIRE CARRYING CURRENT
MAGNETIC FORCES

(a) Opposite poles attract.

(b) Like poles repel.
MAGNETIC FIELD LINES

At each point, the field line is tangent to the magnetic field vector $\vec{B}$. The more densely the field lines are packed, the stronger the field is at that point.

At each point, the field lines point in the same direction a compass would... Therefore, magnetic field lines point away from N poles and toward S poles.
Fig. 27.5 Magnetic Field of Earth

North geographic pole

Compass

Magnetic pole

Magnetic pole

South geographic pole
Force on Particle **Moving** in Magnetic Field

**WILL DEFINE MAGNETIC FIELD**

**CONSIDER POSITIVE TEST PARTICLE**

**THEN OBSERVE**
OBSERVATIONS

The magnetic force is proportional to the charge $q$ and speed $v$ of the particle.

The magnitude and direction of the magnetic force depends on the particle velocity and magnitude and direction of the magnetic field.

Force is zero when velocity and field are in same direction.

When velocity and field make angle $\theta$ the force acts in direction perpendicular to both.
The magnetic force on a positive charge is in the direction opposite to the direction of the force on a negative charge moving in the same direction.

If the velocity vector makes an angle θ with the magnetic field, the magnitude of the magnetic force is proportional to sin θ.

ALL OF THE OBSERVATIONS CAN BE SUMMARIZED WITH THE FOLLOWING EQUATION.
$F = qvB \sin \theta$

Direction is by right hand rule.

Point fingers of right hand in direction of first vector.
Rotate fingers to direction of second vector.
Thumb points in direction of $F$. 
**Right-hand rule** for the direction of magnetic force on a positive charge moving in a magnetic field:

1. Place the \( \vec{v} \) and \( \vec{B} \) vectors tail to tail.

2. Imagine turning \( \vec{v} \) toward \( \vec{B} \) in the \( \vec{v}-\vec{B} \) plane (through the smaller angle).

3. The force acts along a line perpendicular to the \( \vec{v}-\vec{B} \) plane. Curl the fingers of your right hand around this line in the same direction you rotated \( \vec{v} \). Your thumb now points in the direction the force acts.

\[ \vec{F} = q\vec{v} \times \vec{B} \]

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**FORCE ON WIRE CARRYING CURRENT IN MAGNET FIELD.**
WIRE WITH CURRENT – CHARGES MOVING

CHARGES MOVING IN MAGNETIC FIELD WILL HAVE FORCE.

THEREFORE WILL BE FORCE ON WIRE.

FORCE ON WIRE
\[ F = IlB \sin \theta \]

Fingers in direction of first vector, \( I \).

Rotate fingers in direction of second vector, \( B \).

Thumb points in direction of Force.
Question

Which orientation for battery?
MAGNETIC FLUX

Just like Electric Flux but with magnetic field.

\[ \Phi_B = B A \cos \phi \]
Remember Gauss’s Law – the electric flux through a closed surface is equal to $\frac{Q_{en}}{\varepsilon_0}$.

$$\sum EA_{\perp} = 4\pi k Q_{encl}$$

For the magnetic case:

The magnetic flux through a closed surface is equal to zero.

$$\sum BA_{\perp} = 0$$

No magnetic monopoles!
MAGNETIC FIELD vs ELECTRIC FIELD

The electric force is always in the direction of the electric field, whereas the magnetic force is perpendicular to the magnetic field.

The electric force acts on a charged particle independent of the particle’s velocity, whereas the magnetic force acts on a charged particle only when the particle is in motion.

The electric force does work in displacing a charged particle, whereas the magnetic force associated with a steady magnetic field does no work when the particle is displaced.
When charged particle moves magnetic field can change velocity direction but not magnitude (speed) of particle.
MAGNETIC FIELD NEAR WIRE CARRYING CURRENT.
Therefore for current loop:
Force and Torque on Loop

Magnetic moment of loop:

\[ \text{Magnetic Moment} = \text{Current} \times \text{Area} \]

\[ \mu = IA \]

If there are \( n \) loops together:

\[ \mu = nIA \]
Loop for book example:

(a)

The two pairs of forces acting on the loop cancel, so no net force acts on the loop.

However, the forces on the \( a \) sides of the loop (\( \vec{F} \) and \( -\vec{F} \)) produce a torque \( \tau = (IBa)(b \sin \phi) \) on the loop.

\( \phi \) is the angle between a vector normal to the loop and the magnetic field.

Net force is zero.
But will rotate.

Magnetic moment of loop:

\[ \mu = Iab \]

Torque to rotate loop:

\[ \tau = \mu Bs \sin \phi \]
Magnets in magnetic field.

Magnets have magnetic moment:

(b) In a bar magnet, the magnetic moments are aligned.

Magnetic moment wants to be aligned with B.
(c) A magnetic field creates a torque on the bar magnet that tends to align its dipole moment with the $\mathbf{B}$ field.

\begin{align*}
\tau &= \mu B \sin \phi
\end{align*}
Which direction will wire move?
MAGNETIC DEVICES

Motor:

(a) Brushes are aligned with commutator segments.

- Current flows into the red side of the rotor and out of the blue side.
- Therefore the magnetic torque causes the rotor to spin counterclockwise.
(b) Rotor has turned 90°.

- Each brush is in contact with both commutator segments, so the current bypasses the rotor altogether.
- No magnetic torque acts on the rotor.
(c) Rotor has turned $180^\circ$.

- The brushes are again aligned with commutator segments. This time the current flows into the blue side of the rotor and out of the red side.
- Therefore the magnetic torque again causes the rotor to spin counterclockwise.
Hall Effect

(a) Negative charge carriers (electrons)

The charge carriers are pushed toward the top of the strip ...

... so point $a$ is at a higher potential than point $b$. 
Bainbridge’s Mass Spectrometer

Velocity selector selects particles with speed $v$.

Particle detector

Magnetic field separates particles by mass; the greater a particle’s mass, the larger is the radius of its path.
SOURCES OF MAGNETIC FIELD

BASIC CONCEPTS

Magnetic field produced by moving charge.

Magnetic field of current element.

Ampere’s Law

Biot Savart Law
Moving Charge

A moving charge produces a magnetic field. The field will be perpendicular to the direction of motion of the charge.

(a) Perspective view

**Right-hand rule for the magnetic field due to a positive charge moving at constant velocity:**
Point the thumb of your right hand in the direction of the velocity. Your fingers now curl around the charge in the direction of the magnetic field lines. (If the charge is negative, the field lines are in the opposite direction.)

For these field points, $\mathbf{r}$ and $\mathbf{v}$ both lie in the beige plane, and $\mathbf{B}$ is perpendicular to this plane.

For these field points, $\mathbf{r}$ and $\mathbf{v}$ both lie in the gold plane, and $\mathbf{B}$ is perpendicular to this plane.
Magnitude of magnetic field will be

Proportional to charge $q$

Proportional to $\frac{1}{r^2}$

Proportional to speed $v$

Proportional to $\sin \phi$

$$B = \frac{\mu_0}{4\pi} \frac{qv \sin \phi}{r^2}$$
Look at figure and note direction of field vector.

Field is always perpendicular to the direction of motion and a line from the charge to the point where we measure the field.

Therefore

\[ \vec{B} = \frac{\mu_0}{4\pi} \frac{q(vr)}{r^2} \]

\( \vec{r} \) is a unit vector. It has magnitude 1.
A wire carrying current has moving charge so a wire will produce a magnetic field.

(a) Perspective view

**Right-hand rule for the magnetic field due to a current element:** Point the thumb of your right hand in the direction of the current. Your fingers now curl around the current element in the direction of the magnetic field lines.

For these field points, \( \hat{r} \) and \( d\vec{l} \) both lie in the beige plane, and \( d\vec{B} \) is perpendicular to this plane.

For these field points, \( \hat{r} \) and \( d\vec{l} \) both lie in the gold plane, and \( d\vec{B} \) is perpendicular to this plane.

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Therefore similar to the argument for a moving charge we have the field for a section of wire carrying current:

$$\Delta \vec{B} = \frac{\mu_0}{4\pi} \frac{I(\Delta l \sin \phi)}{r^2}$$

This equation is the **Biot-Savart Law**.

It can be used to find the magnetic field of wires in various shapes.
BIOT-SAVART LAW

1. The vector $dB$ is perpendicular both to $dl$ (which is the direction of the current) and to the unit vector $r$ directed from the element to the point $P$.

2. The magnitude of $dB$ is inversely proportional to $r^2$, where $r$ is the distance from the element to the point $P$.

3. The magnitude of $dB$ is proportional to the current and to the length $dl$ of the element.

4. The magnitude of $dB$ is proportional to $\sin\phi$, where $\phi$ is the angle between the vectors $dl$ and $r$. 
Ampere’s Law

Ampere’s law is a simpler way of obtaining the magnetic field if the geometry is right.

The law is

$$\sum B_\parallel \Delta s = \mu_0 I_{\text{enclosed}}$$

This means that if we sum up all products of the components of B parallel to the vector representing the segments of length of the current carrying wire it will equal to $\mu_0$ times the net current enclosed by the path.
Can be used to find $B$ if geometry is right. Otherwise have to use Biot-Savart Law.

Find the magnetic field at $P$.

Must use Biot-Savart Law.
Find the Magnetic Field inside \( (r > R) \) of a conducting cylinder.
We can use Ampere’s Law because of symmetry.

Choose a circular path with radius $r$ where $r > R$.

Ampere’s Law

$$\sum B_\parallel \Delta l = \mu_0 I_{\text{enclosed}}$$

Every place on the circular path $B$ is parallel to the segment.

$$\sum B_\parallel \Delta l = \mu_0 I_{\text{enclosed}}$$
\[ \sum \Delta l = 2\pi r \]

Therefore

\[ B2\pi r = \mu_0 I_{\text{enclosed}} \]

And

\[ I_{\text{enclosed}} = I \]

So

\[ B = \frac{\mu_0 I}{2\pi r} \]
Same problem but for \( r < R \).

Same procedure but need only current enclosed by \( \pi r^2 \).
To find $I_{\text{enclosed}}$ need current density, $j$.

\[ j = \frac{\text{current}}{\text{area}} = \frac{I}{\pi R^2} \]

Current enclosed

\[ I_{\text{enclosed}} = j \times \text{area enclosed} \]

\[ I_{\text{enclosed}} = j \pi r^2 \]

\[ \sum B_{\parallel} \Delta l = \mu_0 \frac{I}{\pi R^2} \pi r^2 \]
\[ B 2\pi r = \mu_0 \frac{I}{\pi R^2} \pi r^2 \]

\[ B 2\pi r = \frac{\mu_0 I}{R^2} r^2 \]

\[ B = \frac{\mu_0 I}{2\pi R^2} r \]
Magnetic Field of Solenoid

\[ B = \mu_0 nI \]

\[ n = \text{number of turns per length} \]
Magnetic Field of a Toroidal Solenoid

The magnetic field is confined entirely to the space enclosed by the windings.

Current out of page

Current into page

Cross-sectional view

$$B = \frac{\mu_0 NI}{2\pi r}$$