CHAPTER 17
ELECTRIC CHARGE AND
ELECTRIC FIELD
TWO BASIC CONCEPTS:

COULOMB’S LAW

\[ F = k \frac{Q_1 Q_2}{r^2} \]

ELECTRIC FIELD

\[ E = \frac{F}{q} \]
(a) Neutral lithium atom (Li):
- 3 protons (3+)
- 4 neutrons
- 3 electrons (3−)

Electrons equal protons:
Zero net charge
IONS – are charged

- Protons (+)
- Neutrons
- Electrons (-)

(b) Positive lithium ion (Li⁺):
- 3 protons (3+)
- 4 neutrons
- 2 electrons (2−)

Fewer electrons than protons:
Positive net charge
SOME MATERIALS –

e’s move easily – conductors
  metals

SOME MATERIALS –

e’s don’t move easily –
  insulators
    glass
    wood
FORCES ON CHARGED OBJECTS

LIKE CHARGES REPEL

UNLIKE CHARGES ATTRACT

(b) Interactions between point charges

Charge of the same sign repel.

\[ F_{1\text{ on }2} = -F_{2\text{ on }1} \]

\[ F_{1\text{ on }2} = F_{2\text{ on }1} = k \frac{|q_1 q_2|}{r^2} \]

Charge of opposite sign attract.
USE COULOMB’S LAW

\[ F = k \frac{Q_1 Q_2}{r^2} \]

Example

Two point charges \( q_1 = 25 \text{ nC} \) and \( q_2 = -75 \text{ nC} \) are separated by a distance of 3.0 cm. Find the magnitude and direction of the electric force that \( q_1 \) exerts on \( q_2 \).
(a) The two charges

\[ F = k \frac{Q_1 Q_2}{r^2} \]

\[ = (9 \times 10^9) \frac{(+25 \times 10^{-9})(-75 \times 10^{-9})}{(0.030m)^2} \]

\[ = 0.01875N \]

Attractive Force
Also

Newton’s third law – if body A exerts a force on body B, then body B exerts an equal and opposite force on body A.
ELECTRIC FIELD

Electric force per unit charge.

\[ E = \frac{F}{q} \]

Or

\[ E = \frac{F}{q_{test}} \]
In direction that positive charge would move.

For more than one charge the total electric field equals the vector sum of all electric fields due to each charge.
For two charges $q_1$ and $q_{\text{test}}$

$$F = k \frac{q_1 q_{\text{test}}}{r^2}$$

So

$$E = \frac{F}{q_{\text{test}}} = k \frac{q_1 q_{\text{test}}}{r^2} q_{\text{test}}$$

Therefore
\[ E = k \frac{q_1}{r^2} \]

Or

\[ E = k \frac{q}{r^2} \]
ALSO

\[ k = \frac{1}{4\pi \varepsilon_0} \]

Where

\[ \varepsilon_0 = 8.854 \times 10^{-12} \, C^2/\text{Nm}^2 \]

Therefore can write

\[ F = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} \]
And

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} \]
ELECTRIC FIELD LINES

1. IN DIRECTION OF FIELD (POSITIVE TEST CHARGE MOVES ALONG LINE.)

2. NUMBER OF LINES PROPORTIONAL TO ELECTRIC FIELD.

3. ELECTRIC FIELD LINES START ON POSITIVE CHARGE AND END ON NEGATIVE CHARGE.
(a) A single positive charge

Field lines always point away from (+) charges and toward (−) charges.
(b) Two equal and opposite charges (a dipole)

Field lines always point away from (+) charges and toward (−) charges.

At each point in space, the electric field vector is tangent to the field line passing through that point.
Example (continued)

Find the electric field 3.0 cm from an electric charge

$q_1 = +25 \text{ nC.}$

$r = 3 \text{ cm}$
What is $E$ at distance $r$ due to $q_1$

\[
E = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} = \frac{1}{4\pi \varepsilon_0} \frac{25 \times 10^{-9}}{0.03^2}
\]

\[
E = 2.5 \times 10^5 \frac{N}{C}
\]

DIRECTION?

Away from positive $q_1$
If we place a charge $-75\text{nC}$ at a point 3 cm away from $q_1$ what is the force on this charge?

(a) The two charges
The electric field at $q_2$ is 

$$2.5 \times 10^5 \frac{N}{C}$$

Therefore

$$F = qE$$

$$= (-75 \times 10^{-9} C)(2.5 \times 10^5 \frac{N}{C})$$

$$= 0.01875 N$$

WE OBTAINED FORCE IN TWO WAYS
Example- Field from two charges

$Q_1 = -50 \mu\text{C}$ at $x=0.52\text{m}, y=0$

$Q_2 = 50 \mu\text{C}$ at $x=0, y=0$ and

find $E$ at point $A$ ($x=0, y=0.3\text{m}$).
\[ E_{A1} = \frac{k(-50 \times 10^{-6} \text{C})}{(0.6 \text{m})^2} \]

\[ = -1.25 \times 10^6 \frac{\text{N}}{\text{C}} \]

(towards \( Q_1 \))

\[ E_{A2} = \frac{k(50 \times 10^{-6} \text{C})}{(0.3 \text{m})^2} \]

\[ = 5 \times 10^6 \frac{\text{N}}{\text{C}} \]

(away from \( Q_2 \))
\[ E_{Ax} = E_{A1} \cos 30^0 \]
\[ = 1.1 \times 10^6 \text{ N/C} \]
\[ E_{Ay} = E_{A2} - E_{A1} \sin 30^0 \]
\[ = 4.4 \times 10^6 \, N/C \]

\[ E_A = \sqrt{E_{Ax}^2 + E_{Ay}^2} \]
\[ = \sqrt{(1.1 \times 10^6)^2 + (4.4 \times 10^6)^2} \]

\[ E_A = 4.5 \times 10^6 \, N/C \]
\[ \tan \phi = \frac{E_{Ay}}{E_{Ax}} = \frac{4.4 \times 10^6 \text{N}/\text{C}}{1.1 \times 10^6 \text{N}/\text{C}} = 4 \]

\[ \phi = 76^0 \]
NOW CALCULATE THE FORCE ON A CHARGE $Q_3 = 50\mu C$ PLACED AT POINT A.

\[ F = qE = (50 \times 10^{-6} C) 4.5 \times 10^6 \frac{N}{C} \]

\[ = 225 N \]

WHAT DIRECTION?
ANYWHERE NEAR AN INFINITE SHEET OF CHARGE THE ELECTRIC FIELD WILL BE

\[ \frac{\sigma}{2\varepsilon_0} \]
GAUSS’S LAW

TWO BASIC CONCEPTS

ELECTRIC FLUX

AND

GAUSS’S LAW

ELECTRIC FLUX
Consider a number of guns at the left shooting to the right.

Flux of bullets is number of bullets times area of sheet perpendicular to bullets.

\[
FLUX_{BULLETS} = N_{BULLETS} \times A_{SHEET}
\]

Turn sheet on edge.
Flux now near zero since area of sheet is parallel to bullets.

Turn sheet to angle.
Flux somewhere between previous two values.

Assign area vector to sheet.
Flux will be

$$\Phi_{Bullets} = N_{Bullets}A \cos \varphi$$

OR THINK ABOUT FLUX THE WAY YOUR BOOK DOES.
(a) A wire rectangle in a fluid
(b) The wire rectangle tilted by an angle $\phi$

$$A_\perp = A \cos \phi$$

$\phi$

$\vec{A}$

$\vec{v}$
FOR ELECTRIC FLUX

\[ \phi = EA \cos \phi \]
GAUSS’S LAW

We will state the law then work some examples. After that we will do an example to justify the law.

Statement of Gauss’s law:

The flux through a closed surface is equal to the net charge enclosed by the surface divided by $\varepsilon_0$.

Equation:

$$\sum EA_{\perp} = 4\pi k Q_{encl}$$
\[ \sum EAcos\varphi = 4\pi kQ_{encl} \]

\[ k = \frac{1}{4\pi \varepsilon_0} \]

Therefore

\[ \sum EAcos\varphi = 4\pi \frac{1}{4\pi \varepsilon_0} Q_{encl} = \frac{Q_{encl}}{\varepsilon_0} \]

Consider a conducting sphere of radius R with charge q on the surface.
Draw a Gaussian surface around this sphere with radius r.

Apply Gauss’s Law
\[ \sum E A \cos \varphi = \frac{Q_{encl}}{\varepsilon_0} \]

\[ Q_{en} = q \]

\[ E(4\pi r^2) = \frac{q}{\varepsilon_0} \]

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} \]

The same equation we had for a point charge.
Example

Positive electric charge $Q$ is distributed uniformly throughout the volume of an insulating sphere with radius $R$.

a. Find the electric field at point $p$ where $r < R$.

b. Find the electric field at point $p$ where $r > R$. 
\[ E(R) = \frac{1}{4\pi \varepsilon_0} \frac{Q}{R^2} \]

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{qr}{R^3} \]

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \]
Part a. Radius of Gaussian Surface = r

Gauss’s Law

\[ \sum EA \cos \phi = \frac{Q_{encl}}{\varepsilon_0} \]

Need charge enclosed by surface.

Charge density \[ \rho = \frac{\text{Total Charge}}{\text{Total Volume}} = \frac{Q}{\frac{4}{3}\pi R^3} \]

Charge enclosed by Gaussian Surface

\[ Q_{en} = \rho \times \text{volume enclosed} \]
\[ Q_{en} = \rho \left( \frac{4}{3} \pi r^3 \right) = \frac{Q}{\frac{4}{3} \pi R^3} \left( \frac{4}{3} \pi r^3 \right) = \frac{r^3}{R^3} Q \]

\[ \sum E A \cos \varphi = \frac{r^3}{R^3} \frac{Q}{\varepsilon_0} \]

\[ E(4\pi r^2) = \frac{r^3}{R^3} \frac{Q}{\varepsilon_0} \]
$E = \frac{r^3}{R^3} \frac{Q}{\varepsilon_0}$

$E = \frac{1}{4\pi \varepsilon_0} \frac{Qr}{R^3}$

For $r < R$
Part b.

Draw Gaussian Surface outside sphere.

\[ \sum EA \cos \varphi = \frac{Q_{encl}}{\varepsilon_0} \]

\[ Q_{en} = Q \]

Same as before

\[ E(4\pi r^2) = \frac{Q}{\varepsilon_0} \]

\[ E = \frac{1}{4\pi \varepsilon_0 r^2} \frac{Q}{r^2} \]
The electric field \( E(R) \) at a distance \( R \) from a uniformly charged spherical insulator is given by:

\[
E(R) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2}
\]

The electric field \( E \) at a distance \( r \) from a point charge \( Q \) is given by:

\[
E = \frac{1}{4\pi\varepsilon_0} \frac{Qr}{R^3}
\]