EXAM 1. Friday, February 28, 2014

Problem 1. *Name*
Write down your name. Clearly. In block letters!

Problem 2. *1987-Fall-SM-G-4*
Consider a cyclic engine operating with one mole of an ideal monoatomic gas in the cycle $a \to b \to c \to d \to a$, where V_a and T_a are the volume and the temperature of the gas in point a.

$a \to b$ is isobaric increase of temperature from T_a to $2T_a$

$b \to c$ isothermal expansion to the volume $3V_a$

$c \to d$ decrease of temperature back to T_a at constant volume

$d \to a$ isothermal compression back to the volume V_a

All processes are reversible.

a. Calculate the efficiency of this engine and compare it to the maximum possible efficiency for an engine operating between T_a and $2T_a$.

b. What is the net entropy change of the gas in one cycle?

c. What is the net change of the energy of the gas in one cycle?

d. What is the net change of the entropy of the hot thermal bath during one cycle, if all processes are reversible?

e. What is the net change of the entropy of the cold thermal bath during one cycle, if all processes are reversible?

Problem 3. *1998-Spring-SM-G-4*
Consider a soap film supported by a wire frame of fixed length l along one direction and of varying length x along the other direction. Because of surface tension σ, there is a force $2\sigma l$ tending to contract the film. Take $\sigma(T, x) = \sigma_0 - \alpha T$, where σ_0 and α are independent of T and x.

1. In one sentence, explain why the force is $2\sigma l$, rather than σl.
2. Express the energy change dE of the film due to work δW associated with the surface tension and heat δQ absorbed by the film through the atmosphere.
3. Calculate the work W done on the film when it is stretched at a constant temperature T_0 from length 0 to x.
4. Calculate the entropy change of the film when it is stretched at constant temperature T_0 from length 0 to x.

5. Calculate the change in energy $\Delta E = E(x) - E(0)$ when the film is stretched at constant temperature T_0 from length 0 to x.

Problem 4. *1998-Spring-SM-G-5*
Consider a cyclic engine operating with one mole of an ideal monoatomic gas in the following reversible cycle:

$a \to b$: expansion at constant pressure, the temperature going from T_a to $3T_a$.

$b \to c$: expansion at constant temperature $3T_a$, the volume going to $4V_a$.

$c \to d$: cooling at constant volume $4V_a$, the temperature going to T_a.

$d \to a$: compression at constant temperature, the volume going from $4V_a$ to V_a.

1. Sketch the cycle on a $P - V$ diagram.
2. Find the entropy change and the change in internal energy of the gas for each part of the cycle. Find the net entropy change and the net change in internal energy over the full cycle.
3. Calculate the thermodynamic efficiency of this engine and compare it to the ideal efficiency of an engine operating between T_a and $3T_a$. Assume no irreversible process occur.

EXAM 2. Friday, April 4, 2014

Problem 1. *Name. 1pt*
Write down your name. Clearly. In block letters!

Problem 2. *1985-Fall-SM-G-5*
A $3D$ harmonic oscillator has mass m and the same spring constant k for all three directions. Thus, its quantum-mechanical energy levels are given by

$$E = \hbar\omega(n_1 + n_2 + n_3 + 3/2)$$

Neglect the zero point energy in what follows.

1. What are the energy E_0 and degeneracy g_0 of the ground sate? Of the first excited state? Of the second excited state?
2. What is the partition function for this system if only the lowest two (ground and the first exited) energy levels are important? When this approximation valid?
3. What is the free energy in this case?
4. What is the entropy in this case?
5. What is the rms fluctuation $\delta E \equiv (\overline{E^2} - (\bar{E})^2)^{1/2}$ in the energy in this case?

Problem 3. *1987-Fall-SM-G-6*

1. Consider a particle which can take one of only two energy levels ($E = 0$ or $E = \epsilon$). Find and sketch as a function of T the average energy $\bar{E}(N, T)$ and the heat capacity $C_V(N, T)$ for a system of N particle. The particles are distinguishable.
2. Suppose that the value of the excited state energy ϵ varies for different particles in a system of N particles. Furthermore, assume that the number of particles with a particular value of the excited state energy between ϵ and $\epsilon + d\epsilon$ is $n(\epsilon)d\epsilon$ and suppose that $n(\epsilon)$ is a constant for all energies between 0 and ϵ_0, and that there is no states with $\epsilon > \epsilon_0$, i.e.
$$n(\epsilon) = \begin{cases} n_0, & \text{for } 0 < \epsilon \le \epsilon_0 \\ 0, & \text{for } \epsilon > \epsilon_0 \end{cases}$$
and $N = \int_0^\infty n(\epsilon)d\epsilon = n_0\epsilon_0$. Find the temperature dependence of the specific heat of these N particles at low temperature, $T \ll \epsilon_0/k_B$.

Problem 4. *1992-Fall-SM-G-5*
In experiments on the absorption spectrum of gases at finite temperature, atoms are always moving either towards or away from the light source with distribution of velocities v_x. As a consequences the frequency of the photon seen by an electron in a Bohr atom is Doppler shifted to a value ν_D according to the classical formula

$$\nu_D = \nu_0(1 + v_x/c).$$

1. Write down the normalized Maxwell distribution for v_x.

2. Determine the distribution function $g(\nu_D)$ for the fraction of gas atoms that will absorb light at frequency ν_D.

3. Determine the fractional linewidth

$$\left(\frac{\Delta\nu}{\nu_0}\right)_{\mathrm{rms}} \equiv \sqrt{\left\langle\frac{(\nu_D-\nu_0)^2}{\nu_0^2}\right\rangle}.$$

4. Estimate the fractional linewidth for a gas of hydrogen ($M \approx 938MeV/c^2$, $1eV/k_B \approx 11600K$) atoms at room temperature.