EXAM 1. Monday, March 4, 2013

Problem 1. Name. 1pt

Write down your name. Clearly. In block letters!

Problem 2. Almost van der Waals. 33pt.

The energy of the gas depends on temperature and volume according to

$$E = f_1(T) + f_2(V),$$

Where f_1 and f_2 are some known/given functions.

a.*(11pt.)* What can you say about temperature dependence of α/β_T?

b.*(11pt.)* If α/β_T is given how the temperature of this gas will change if the gas undergoes the expansion by dV into vacuum?

c.*(11pt.)* If α/β_T is given how the temperature of this gas will change if the gas undergoes the expansion by dV by the adiabatic process?

Problem 3. Almost 1987-Fall-SM-G-4. 33pt.

Consider a cyclic engine operating with one mole of an ideal monoatomic gas in the cycle which consists of three legs $a \to b \to c \to a$. T_a is the temperature of the gas in point a.

$a \to b$ is increase of temperature from T_a to $4T_a$ in such a process that $P = \lambda V$.

$b \to c$ is compression at constant pressure back to V_a.

$c \to a$ is cooling at constant volume back to T_a.

All processes are reversible.

a.*(5pt.)* What is the net entropy change of the gas in one cycle?

b.*(5pt.)* What is the net change of the energy of the gas in one cycle?

c.*(6pt.)* What is the change of entropy of the gas during each leg?

d.*(6pt.)* What work is done by the gas during each leg?

e.*(6pt.)* What is the total heat that the gas obtained/gave up during the full cycle?

f.*(5pt.)* What is the total work done by the gas during the full cycle?

Problem 4. 1988-Fall-SM-G-4. 33pt.

Consider N identical and independent particles of spin 1/2 in a magnetic field so that the energy of each particle is either $+\epsilon$ or $-\epsilon$ depending upon the orientation of the spin: down or up respectively. The probability of the energy $-\epsilon$ is p and of $+\epsilon$ is $q = 1 - p$.

a. *(11pt.)* Find the probability that N_1 spins are up and N_2 are down, with $N_1 + N_2 = N$.

b. *(11pt.)* From this probability find the average or expected value of the total energy in terms of p, ϵ, and N.

c. *(11pt.)* By considering that N_1 and N_2 are continuous rather than discrete variables, find the most likely (i.e., maximal probability) for the values N_1 and N_2 using the approximation that $\log(x!) \approx x \log(x) - x$.

EXAM 2. Monday, April 8, 2013

Problem 1. Name. 1pt

Write down your name. Clearly. In block letters!

Problem 2. Qualifying Exam. 1987 Fall. Graduate level. 33pt

1. *10pt* Consider a particle which can take one of only two energy levels ($E = 0$ or $E = \epsilon$). Find and sketch as a function of T the average energy $\bar{E}(N, T)$ and the heat capacity $C_V(N, T)$ for a system of N particle. The particles are distinguishable.

2. *23pt* Suppose that the value of the excited state energy ϵ varies for different particles in a system of N particles. Futhermore, assume that the number of particles with a particular value of the excited state energy between ϵ and $\epsilon + d\epsilon$ is $n(\epsilon)d\epsilon$ and suppose that $n(\epsilon)$ is a constant for all energies between 0 and ϵ_0, and that there is no states with $\epsilon > \epsilon_0$, i.e.
$$n(\epsilon) = \begin{cases} n_0, & \text{for } 0 < \epsilon \leq \epsilon_0 \\ 0, & \text{for } \epsilon > \epsilon_0 \end{cases}$$
and $N = \int_0^\infty n(\epsilon)d\epsilon = n_0\epsilon_0$. Find the temperature dependence of the specific heat of these N particles at low temperature, $T \ll \epsilon_0/k_B$.

Problem 3. Qualifying Exam. 1992 Fall. Graduate level. 33pt

In experiments on the absorption spectrum of gases at finite temperature, atoms are always moving either towards or away from the light source with distribution of velocities v_x. As a consequences the frequency of the photon seen by an electron in a Bohr atom is Doppler shifted to a value ν_D according to the classical formula

$$\nu_D = \nu_0(1 + v_x/c).$$

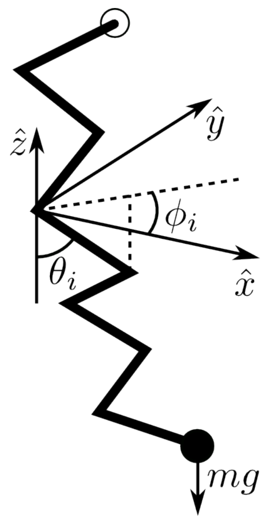
1. *10pt* Write down the normalized Maxwell distribution for v_x.

2. *10pt* Determine the distribution function $g(\nu_D)$ for the fraction of gas atoms that will absorb light at frequency ν_D.

3. *10pt* Determine the fractional linewidth
$$\left(\frac{\Delta\nu}{\nu_0}\right)_{\text{rms}} \equiv \sqrt{\left\langle \frac{(\nu_D - \nu_0)^2}{\nu_0^2} \right\rangle}.$$

4. *3pt* Estimate the fractional linewidth for a gas of hydrogen atoms ($M \approx 938MeV/c^2$, $1eV/k_B \approx 11600K$) at room temperature.

Problem 4. Qualifying Exam. 1999 Spring. Graduate level. 33pt



A polymer is a molecule composed of a long chain of identical molecular units, called monomers. For simplicity, assume the polymer consists of a long chain of N rod-like monomers, each of length l, attached end to end. Assume that the connectors between the monomers are completely flexible so that each monomer can make any angle with respect to its neighbors. One end of the polymer is fixed while the other end is attached to a weight (mg) providing a constant force in the negative $\hat{z}$ direction. Except for an arbitrary constant which can be ignored, the potential energy of any configuration of the polymer can be written as,

$$E(\theta_1, \phi_1, \theta_2, \phi_2, \ldots \theta_N, \phi_N) = \sum_{i=1}^{N} mgl(1 - \cos(\theta_i)),$$

where θ_i, ϕ_i represent angles in the spherical coordinates that the i^{th} monomer makes with the negative $\hat{z}$ direction. Each angle can take all possible values $0 < \theta_i < \pi$ and $0 < \phi_i < 2\pi$.

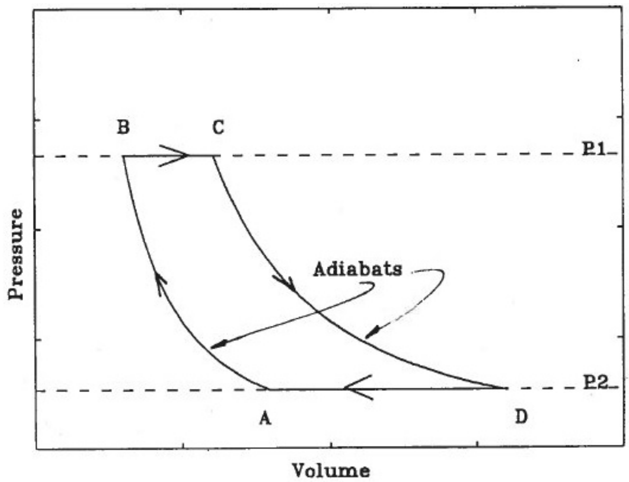
1. *11pt* Determine the classical canonical partition function $Z(T, N)$ for this polymer in contact with a temperature bath at temperature T.

2. *11pt* Determine the thermodynamic energy $U(T, N)$ and entropy $S(T, N)$ of the system. Find the temperature dependence of U and S for very small and very large temperatures.

3. *11pt* If the polymer is isolated from the temperature bath and the mass at the end of the polymer is increased adiabatically from its initial value of m to a final value of $2m$, determine the final temperature of the polymer.

EXAM 3. Final.Tuesday, May. 7, 2013

Problem 1. Qualifying Exam. 1993 Spring. Graduate level. 25pt



A particular ideal gas is characterized by the following equations of state:

$$PV = Nk_BT \qquad \text{and} \qquad U = cNk_BT,$$

where c is a constant. The gas is used as a medium in a heat engine/refrigerator whose cycle an be represented on a $P - V$ diagram as shown in the figure.

1. *6pt* Determine the change in the internal energy of the gas for one complete cycle ($A \to B \to C \to D \to A$).

2. *6pt* Determine the change in the entropy of the gas for one complete cycle ($A \to B \to C \to D \to A$).

3. *6pt* Determine the equation of the adiabats.

4. *7pt* Determine the thermodynamic efficiency of the above heat engine. Express your answer in terms of c, P_1, and P_2.

Problem 2. Qualifying Exam. 1996 Fall. Graduate level. 25pt

Consider a molecular clock that can take on four angular positions $\theta = n(\pi/2)$, for $n = 0, 1, 2, 3$. Its energy is given by

$$E = -A\cos\theta.$$

It is subject to thermal fluctuations at a temperature T.

1. *12pt* Determine the value of $\langle\cos\theta\rangle$. Give the high-temperature and low-temperature limits. Interpret your results physically.

2. *13pt* Determine the value of $\langle\cos^2\theta\rangle$. Give the high-temperature and low-temperature limits. Interpret your results physically.

Problem 3. Qualifying Exam. 1990 Fall. Graduate level. 25pt

Let $\epsilon_{\vec{p}} = \rho(|p_x| + |p_y| + |p_z|)$ be the energy-momentum relation of a conducting electrons in a certain (fictitious) metal, where ρ is a constant with the dimensions of velocity, and $|x|$ denotes the absolute value of x.

1. *5pt* Draw a picture of a typical constant-energy surface $\epsilon_{\vec{p}} = \epsilon$, for the above $\epsilon_{\vec{p}}$.

2. *5pt* Express the Fermi energy ϵ_F as a function of the electron density n for this metal. (Note, that electrons have spin 1/2.)

3. *5pt* Calculate the electronic density of states (per unit volume) as a function of ϵ for this metal. Denote it as $D(E)$.

4. *5pt* Calculate the total energy of N electrons at $T = 0$ in this metal. Express E_0/N in terms of ϵ_F.

5. *5pt* Calculate the heat capacity per electron of this gas at small temperature. Express result in terms of T/ϵ_F.

Problem 4. Qualifying Exam. 1990 Spring. Graduate level. 25pt

A system of ideal Boltzmann gas containing N monoatomic molecules (of mass m each) is trapped in a three-dimensional harmonic potential well $V(x, y, z) = \frac{1}{2}K\,(x^2 + y^2 + z^2)$, so it will form a spherical cloud whose room-mean-square radius $\bar{r}_{\text{rms}} \equiv \left(\langle x^2 + y^2 + z^2 \rangle\right)^{1/2}$ will be the function of temperature. Calculate:

1. *5pt* The (quantum) canonical partition function, (which no longer has volume V as an independent variable due to the trapping potential).

2. *5pt* The free energy $F(N, T)$;

3. *5pt* The total entropy of the cloud $S(N, T)$;

4. *5pt* Take the high-temperature approximation of $S(N, T)$ to the leading order in some small parameter, and use it to obtain the specific heat $C(N, T)$ in this limit to the same order. (Note: the specific heat is not of constant volume, nor of constant pressure, but of constant K, the force constant, and, of course, constant N.) Interpret your result in light of the equipartition theorem.

5. *5pt* Given that $\langle n|x^2|n\rangle = (n+1/2)\hbar\omega_0/K$ for a quantum mechanical harmonic oscillator of mass m and force constant K, calculate $\bar{r}_{\text{rms}}$, and hence the "mean" volume $\bar{V} \equiv \frac{4\pi}{3}\bar{r}^3_{\text{rms}}$, of the above cloud as a function of N and T. What is the smallest $\bar{V}$ at any temperature, and what is the behavior of $\bar{V}$ with respect to T in the high-temperature limit?

THE END!