EXAM 1. Friday, October 4, 2013

Problem 1. ***Name***
Write down your name. Clearly. In block letters!

Problem 2. ***1985-Spring-SM-G-5***

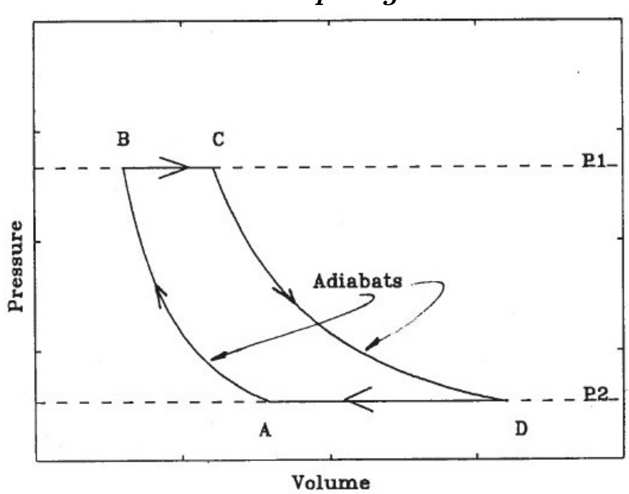
1. Show that for an ideal gas, for a quasistatic adiabatic process
$$\frac{dP}{dV} = -\gamma \frac{P}{V},$$
where $\gamma = C_P/C_V$.

2. Write down the expression for the work done by one mole of a monoatomic ideal gas during an adiabatic expansion from a state (P_1, V_1) to a state with volume V_2.

3. The molar energy of a monoatomic gas which obeys van der Waals' equation of state is given by
$$E = \frac{3}{2}RT - \frac{a}{V},$$
where V is the molar volume at a temperature T and a is a constant of the gas. If initially, one mole of the gas is at the temperature T_1 and occupies volume V_1, and the gas is allowed to expand irreversibly into a vacuum in an isolated container, so that it occupies a total volume V_2, what is the final temperature T_2 of the gas?

4. Calculate the change of the temperature in this process for the ideal gas. Give the reason for your answer explaining concisely the origin of the differences (**In one short(!!!!) sentence.**).

Problem 3. ***1993-Spring-SM-G-1***



A particular ideal gas is characterized by the following equations of state:

$$PV = Nk_BT \qquad \text{and} \qquad U = cNk_BT,$$

where c is a constant. The gas is used as a medium in a heat engine/refrigerator whose cycle an be represented on a $P-V$ diagram as shown in the figure.

1. Determine the change in the internal energy of the gas for one complete cycle ($A \to B \to C \to D \to A$).

2. Determine the equation of the adiabats.

3. Determine the thermodynamic efficiency of the above heat engine. Express your answer in terms of c, P_1, and P_2.

Problem 4. ***1998-Spring-SM-G-4***

Consider a soap film supported by a wire frame of fixed length l along one direction and of varying length x along the other direction. Because of surface tension σ, there is a force $2\sigma l$ tending to contract the film. Take $\sigma(T, x) = \sigma_0 - \alpha T$, where σ_0 and α are independent of T and x.

1. In one sentence, explain why the force is $2\sigma l$, rather than σl.

2. Express the energy change dE of the film due to work dW associated with the surface tension and heat dQ absorbed by the film through the atmosphere.

3. Calculate the work W done on the film when it is stretched at a constant temperature T_0 from length 0 to x.

4. Calculate the entropy change of the film when it is stretched at constant temperature T_0 from length 0 to x.

5. Calculate the change in energy $\Delta E = E(x) - E(0)$ when the film is stretched at constant temperature T_0 from length 0 to x.

EXAM 2. Friday, November 1, 2013

Problem 1. ***Name. 1pt***
Write down your name. Clearly. In block letters!

Problem 2. ***1983-Spring-SM-G-5.***
At $T = 0$, all the N atoms in a crystal occupy a lattice site of a simple cubic lattice with no vacancies. At higher temperature, it is possible for an atom to move from a lattice site to an interstitial site in the center of a cube (the interstitial atom does not have to end up close to vacancy). An atom needs energy ϵ to make this transition.

a. Compute the number of different ways of making n vacancies (and correspondingly fill n interstitial sites) in the lattice.

b. Calculate the entropy of a state with energy $E = n\epsilon$.

c. Calculate the average $\langle n \rangle$ in equilibrium at temperature T.

d. Calculate the free energy of the lattice at temperature T.

Problem 3. ***1992-Fall-SM-G-5***
In experiments on the absorption spectrum of gases at finite temperature, atoms are always moving either towards or away from the light source with distribution of velocities v_x. As a consequences the frequency of the photon seen by an electron in a Bohr atom is Doppler shifted to a value ν_D according to the classical formula

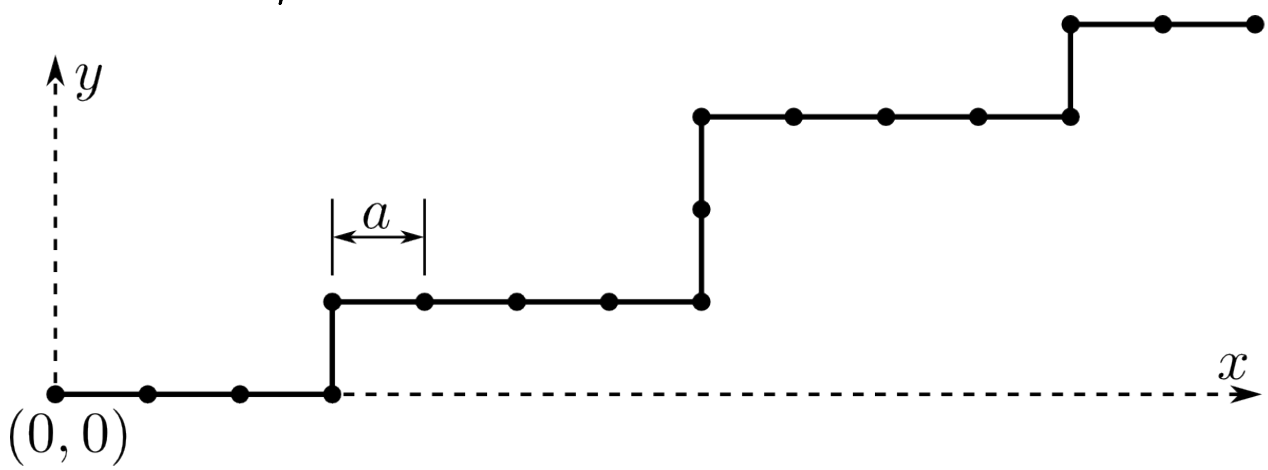
$$\nu_D = \nu_0(1 + v_x/c).$$

1. Write down the normalized Maxwell distribution for v_x.
2. Determine the distribution function $g(\nu_D)$ for the fraction of gas atoms that will absorb light at frequency ν_D.
3. Determine the fractional linewidth

$$\left(\frac{\Delta\nu}{\nu_0}\right)_{\text{rms}} \equiv \sqrt{\left\langle \frac{(\nu_D - \nu_0)^2}{\nu_0^2} \right\rangle}.$$

4. Estimate the fractional linewidth for a gas of hydrogen ($M \approx 938 MeV/c^2$, $1eV/k_B \approx 11600K$) atoms at room temperature.

Problem 4. ***1994-Fall-SM-G-1***



A two dimensional fictitious "polymer" is constructed as follows: It consists of N identical monomers of length a each, joined into a chain. (See figure.) Starting from the left end of the "polymer", which corresponds to the coordinates $x = 0$, $y = 0$, each new monomer added prefers to extend the "polymer" to the right (i.e., to the $+x$ direction) with energy $\epsilon = 0$, or upward (i.e., to the $+y$ direction), with energy $\epsilon = J > 0$. It is given that no monomer added will extend the polymer to the left or downward.

1. A macro state of this "polymer" is characterized by N and m, where m denotes the number of monomers which extend the "polymer" upward. Find the entropy of this macro state as a function of N and m. Assume that both N and m are $\gg 1$, and use Stirling's formula $\ln N! \approx N(\ln N - 1)$ to simplify your result.

2. Write down the free energy of this macro-state of the "polymer", as a function of N, m, and the temperature T.

3. From minimizing this free energy, calculate the equilibrium value of m as a function of N and T.

4. Calculate the equilibrium value of the free energy as a function of N and T, and from which, calculate the specific heat of this "polymer" as a function of N and T. (Note: Simplify your expression for the free energy to something quite simple before calculating specific heat! *Make sure your result has the right dimension, or else you get no credit for this point.*)

5. Show that in the limit $k_B T \gg J$, the specific heat of this "polymer" is inversely proportional to T^2, and obtain the coefficient of proportionality.

EXAM 3. Final. Monday, December 9, 2013

Write down your name. Clearly. In block letters!

Problem 1. ***1983-Fall-SM-G-5***

a. What is the average energy of a system at temperature T with two quantum states, with an energy difference ϵ separating the levels?

b. In a highly disordered solid it is believed that a large number of two-level "tunneling" systems are present with a distribution of energy differences. If the number of these two level systems per unit volume of the solid with energy separations between ϵ and $\epsilon + d\epsilon$, $D(\epsilon)$, is a constant, show that their contribution to the heat capacity of the solid will be linear in temperature.

Problem 2. ***1984-Fall-SM-G-7***

Suppose we have some type of wavelike excitation in a solid which is characterized by a free quasiparticle of spin 3. The total number of quasiparticles in the system is not conserved. Furthermore, the excitation (quasiparticle) obeys the dispersion relation $\omega = Ak^3$, where ω is the angular frequency of the excitation, k is its wave number, and A is a constant of proportionality. If the solid has a volume V and is macroscopic in size, then determine the temperature dependence of both the internal energy and heat capacity in the limit of very low temperatures.

Problem 3. ***1987-Fall-SM-G-4***

Consider a cyclic engine operating with one mole of an ideal monoatomic gas in the cycle $a \to b \to c \to d \to a$, where V_a and T_a are the volume and the temperature of the gas in point a.

$a \to b$ is isobaric increase of temperature from T_a to $2T_a$

$b \to c$ isothermal expansion to the volume $3V_a$

$c \to d$ decrease of temperature back to T_a at constant volume

$d \to a$ isothermal compression back to the volume V_a

All processes are reversible.

a. Calculate the efficiency of this engine and compare it to the maximum possible efficiency for an engine operating between T_a and $2T_a$.

b. What is the net entropy change of the gas in one cycle?

c. What is the net change of the energy of the gas in one cycle?

d. What is the net change of the entropy of the hot thermal bath during one cycle, if all processes are reversible?

e. What is the net change of the entropy of the cold thermal bath during one cycle, if all processes are reversible?

Problem 4. ***2000-Spring-SM-G-4***

A box of volume V contains N molecules of a classical gas. The molecules can either be bound to the walls of the box with binding energy ϵ_b and dispersion relation of a $2D$ gas $\epsilon(p) = (p_x^2 + p_y^2)/2m$, or move freely within the volume of the box with dispersion relation of a $3D$ gas $\epsilon(p) = (p_x^2 + p_y^2 + p_z^2)/2m$.

Find what portion of the molecules are stuck to the surface. Obtain an explicit results in two limiting cases of low and high temperature.

THE END!