EXAM 1. Given as homework. Due the first class after Spring Break.

Problem 1. Fluctuations.

A brick of mass M is sitting on a table at temperature T.

a. How high the brick is hovering over the table?

b. Estimate the numbers.

Problem 2. Rotating gas.

The canonical partition function of a classical, monoatomic, ideal gas in a cylinder rotating with angular velocity ω is given by (see Kubo chapter 2, problem 4)

$$Z = \frac{1}{N!}\left[\pi R^2 L \left(\frac{2\pi m T}{h^2}\right)^{3/2} \frac{e^x - 1}{x}\right]^N$$

where $x = \frac{m\omega^2 R^2}{2T}$.

a. Find the angular momentum M of the rotating gas as a function of temperature and angular velocity.

b. Consider the limits of the obtained expression for M corresponding to very high and very low temperatures. Give the physical interpretation of obtained results. What is the criterion of high and low in this case?

c. How much energy one should supply to heat the gas from a very low temperature T_0 to a very high temperature T_f? Denote the initial angular velocity of the cylinder as ω_0 . Neglect the moment of inertia of the cylinder (vessel) itself.

Problem 3. Quantum oscillator.

A quantum particle of charge q is in the oscillator potential $V(x) = \frac{m\omega^2}{2}x^2$ in $1D$ at temperature T.

a. Find the heat capacity of this system.

b. Find the electric dipole susceptibility of the system. $(d = -(\partial F/\partial E)_{V,N,T}$, $\chi = (\partial d/\partial E)_{V,N,T}$

c. For the oscillator state ψ_n in the presence of electric field E calculate $x_n = \langle \psi_n | \hat{x} | \psi_n \rangle$, and then average displacement $\bar{x} = \sum_n x_n w_n$.

Problem 4. N bricks

You have N identical bricks with temperatures T_i, $i = 1 \dots N$.

a. What is the highest temperature you can give to one of the bricks.

b. If all of the initial temperatures are in a small interval $T_0 < T_i < T_0 + \Delta T$, where $\Delta T \ll T_0$, find the highest temperature of part a. for $N \to \infty$.

Problem 5. String

A $3D$ string of length L has a temperature dependent tension $f(T)$.

a. What is the average value of the amplitude a_k^α of the kth harmonic of a small deformation of the string?

b. What is $\langle a_k^\alpha a_{k'}^\beta \rangle$ for two harmonics k and k'?

c. Calculate $\left\langle e^{\vec{n}\vec{a}_k} \right\rangle$.

EXAM 2. Friday, Apr. 13, 2012

Problem 1. Name. 1pt

Write down your name. Clearly. In block letters!

Problem 2. Relativistic Fermions. 33pt

Relativistic spinless non interacting $2D$ fermions have the dispersion relation $\epsilon_p = \sqrt{m^2c^4 + c^2p^2}$. If the concentration of the fermions is n and temperature T

a. Calculate the density of states $\nu(\epsilon)$.

b. Calculate the Fermi momentum p_F of the gas.

c. Calculate the Fermi energy ϵ_F of the gas.

d. Write down the implicit equation of state.

e. At what temperature the relativistic nature of the fermions is important?

Problem 3. A chain of sticks. A polymer. 33pt

A chain of N massless sticks of length l each is attached by one end to a point in space. The sticks can freely rotate in the joints. There is a massless charge q on the other end of the chain. There is a uniform electric field $\mathcal{E}$ along the z axis in the space. The chain is coupled to and is in thermal equilibrium with the heat bath of temperature T.

a. Determine the Free energy, Entropy, and Energy of the system. What is the energy at $T = 0$ and at $T \to \infty$.

b. Determine the average z coordinate of the charge q. Determine the fluctuations of the z component of the charge q.

c. If the polymer/chain is isolated from the thermal bath and Electric field is increased adiabatically from $\mathcal{E}$ to $2\mathcal{E}$, determine the final temperature.

d. (**extra 5 points**) Determine average square of the length of the polymer.

Problem 4. Perturbation. 33pt

The gas of N identical particles is in a volume V at temperature T. The particles interact weakly through two-body potentials

$$U_{ij} = De^{-(\vec{r}_i - \vec{r}_j)^2/a^2},$$

where D and a are constants, and $\vec{r}_i$s are particles' coordinates. Determine the leading correction in D to the (Helmholtz) Free energy F of the system at high temperatures, and the resulting change in the pressure of the gas.

EXAM 3. Final. Friday, May. 4, 2012

Problem 1. Your name. 1pt

Write your name. Clearly.

Problem 2. Vacancies in crystal. 33pt

At $T = 0$, all the N atoms in a crystal occupy a lattice site of a simple cubic lattice with no vacancies. At higher temperature, it is possible for an atom to move from a lattice site to an interstitial site in the center of a cube (the interstitial atom does not have to end up close to vacancy). An atom needs energy ϵ to make this transition.

a. Compute the number of different ways of making n vacancies (and correspondingly fill n interstitial sites) in the lattice.

b. Calculate the entropy of a state with energy $E = n\epsilon$.

c. Calculate the average $\langle n \rangle$ in equilibrium at temperature T.

d. Calculate the free energy of the lattice at temperature T.

Problem 3. Fictitious metal. 33pt

In a certain $2D$ fictitious metal the electron's (spin=1/2) dispersion relation is $\epsilon = v\,(|p_x| + |p_y|)$, where v is a constant.

a. Draw a picture of a typical constant energy surface/line.

b. Find ϵ_F for a given electron's density.

c. Calculate the electronic density of states $\nu(\epsilon)$.

d. Calculate the total energy E_0 of N electrons at $T = 0$ in this metal. Express E_0/N in terms of ϵ_F.

e. At small temperature $T \ll \epsilon_F$ find the energy per particle (E/N) and chemical potential μ as functions of temperature. Express the answer through ϵ_F and T.

Problem 4. Work. 33pt

Consider a cyclic engine operating with one mole of an ideal monoatomic gas in the cycle $a \to b \to c \to d \to a$, where V_a and T_a are the volume and the temperature of the gas in point a.

$a \to b$ is isobaric increase of temperature from T_a to $2T_a$

$b \to c$ isothermal expansion to the volume $3V_a$

$c \to d$ decrease of temperature back to T_a at constant volume

$d \rightarrow a$ isothermal compression back to the volume V_a

All processes are reversible.

a. Calculate the maximal efficiency of this engine.

b. What is the net entropy change of the gas in one cycle?

c. What is the net change of the energy of the gas in one cycle?

d. What is the net change of the entropy of the hot thermal bath during one cycle, if all processes are reversible?

e. What is the net change of the entropy of the cold thermal bath during one cycle, if all processes are reversible?