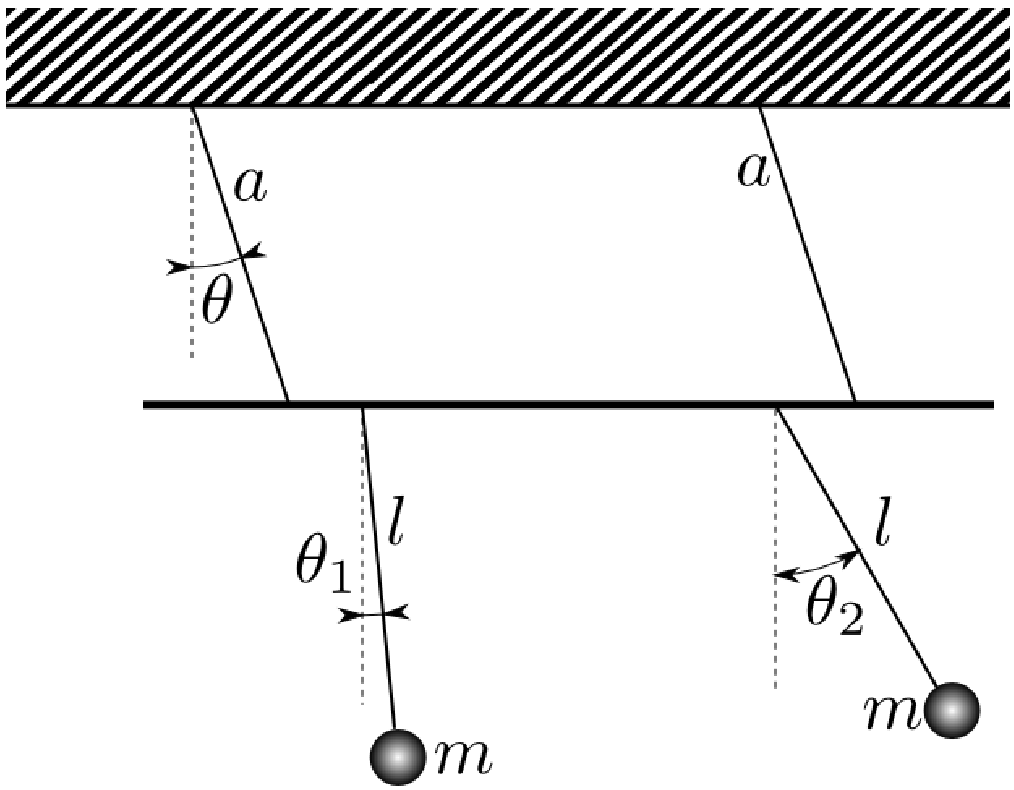
EXAM 1. Thursday, March 23, 2017, 2:20-3:35pm

Problem 1. *1990-Fall-CM-G-4*
A bar of negligible weight is suspended by two massless rods of length a. From it are hanging two identical pendula with mass m and length l. All motion is confined to a plane. Treat the motion in the small oscillation approximation. (Hint: use θ, θ_1, and θ_2 as generalized coordinates.)

1. Find the normal mode frequencies of the system.
2. Find the eigenvector corresponding to the lowest frequency of the system.
3. Describe physically the motion of the system oscillating at its lowest frequency.

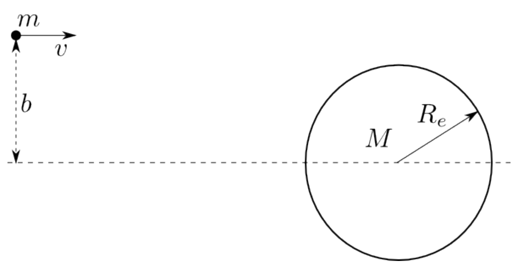


Problem 2. *1993-Spring-CM-G-4.jpg*
Because of the gravitational attraction of the earth, the cross section for collisions with incident asteroids or comets is larger than πR_e^2 where R_e is the physical radius of the earth.

1. Write the Lagrangian and derive the equations of motion for an incident object of mass m. (For simplicity neglect the gravitational fields of the sun and the other planets and assume that the mass of the earth, M is much larger than m.)
2. Calculate the effective collisional radius of the earth, R, for an impact by an incident body with mass, m, and initial velocity v, as shown, starting at a point far from the earth where the earth's gravitational field is negligibly small. Sketch the paths of the

incident body if it starts from a point 1) with $b < R_e$ 2) with $b \gg R_e$, and 3) at the critical distance R. (Here b is the impact parameter.)

3. What is the value of R if the initial velocity relative to the earth is $v = 0$? What is the probability of impact in this case?



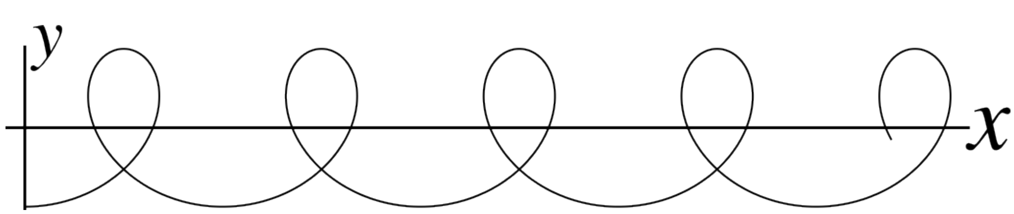
Problem 3. *1997-Spring-CM-G-4.jpg*

The curve illustrated below is a parametric two dimensional curve (not a three dimensional helix). Its coordinates $x(\tau)$ and $y(\tau)$ are

$$x = a\sin(\tau) + b\tau$$
$$y = -a\cos(\tau),$$

where a and b are constant, with $a > b$. A particle of mass m slides without friction on the curve. Assume that gravity acts vertically, giving the particle the potential energy $V = mgy$.

1. Write down the Lagrangian for the particle on the curve in terms of the single generalized coordinate τ.
2. From the Lagrangian, find p_τ, the generalized momentum corresponding to the parameter τ.
3. Find the Hamiltonian in terms of the generalized coordinate and momentum.
4. Find the two Hamiltonian equations of motion for the particle from your Hamiltonian.



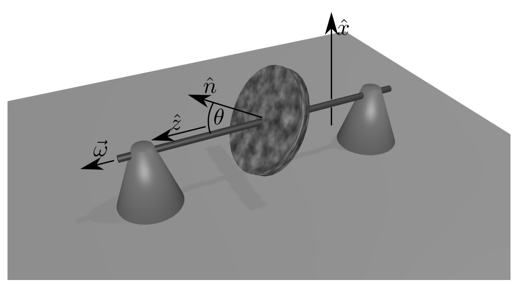
EXAM 2. Final. Tuesday, May 9, 2017, 1:00-3:00pm

Problem 1. *1984-Fall-CM-G-5*
A ladder of length L and mass M rests against a smooth wall and slides without friction on wall and floor. Initially the ladder is at rest at an angle $\alpha = \alpha_0$ with the floor. (For the ladder the moment of inertia about an axis perpendicular to and through the center of the ladder is $\frac{1}{12}ML^2$).

1. Write down the Lagrangian and Lagrange equations for $\alpha(t)$.
2. Find the first integral of motion in the angle α.
3. Determine the force exerted by the wall on the ladder as a function of α.
4. Determine the angle α_c at which the ladder leaves the wall.

Problem 2. *1985-Fall-CM-G-6*
A disk is rigidly attached to an axle passing through its center so that the disc's symmetry axis $\hat{n}$ makes an angle θ with the axle. The moments of inertia of the disc relative to its center are C about the symmetry axis $\hat{n}$ and A about any direction $\hat{n}'$ perpendicular to $\hat{n}$. The axle spins with constant angular velocity $\vec{\omega} = \omega\hat{z}$ ($\hat{z}$ is a unit vector along the axle.) At time $t = 0$, the disk is oriented such that the symmetry axis lies in the $X - Z$ plane as shown.

1. What is the angular momentum, $\vec{L}(t)$, expressed in the space-fixed frame.
2. Find the torque, $\vec{\tau}(t)$, which must be exerted on the axle by the bearings which support it. Specify the components of $\vec{\tau}(t)$ along the space-fixed axes.



Problem 3. *1989-Fall-CM-G-5*
Consider a particle of mass m interacting with an attractive central force field of the form

$$V(r) = -\frac{\alpha}{r^4}, \qquad \alpha > 0.$$

The particle begins its motion very far away from the center of force, moving with a speed v_0.

1. Find the effective potential V_{eff} for this particle as a function of r, the impact parameter b, and the initial kinetic energy $E_0 = \frac{1}{2}mv_0^2$. (Recall that V_{eff} includes the centrifugal effect of the angular momentum.)

2. Draw a qualitative graph of V_{eff} as a function of r. (Your graph need not show the correct behavior for the special case $b = 0$.) Determine the value(s) of r at any special points associated with the graph.

3. Find the cross section for the particle to spiral in all the way to the origin.

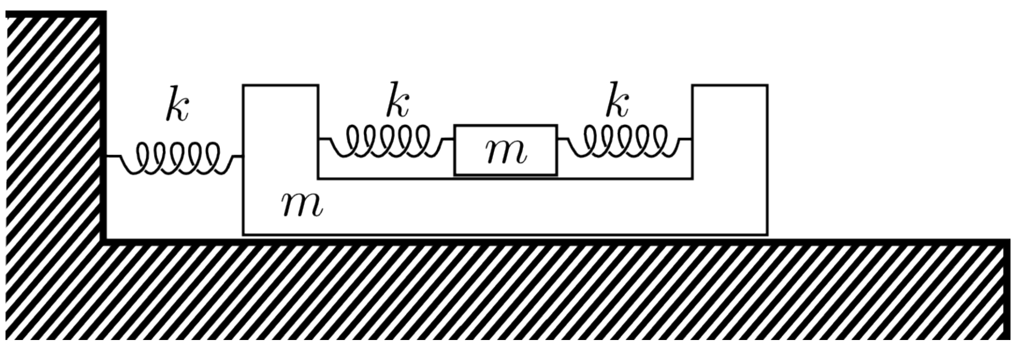
Problem 4. *1998-Fall-CM-U-3*

The two "blocks" in the figure below are frictionless and have the same mass m. The larger block is sitting on a horizontal, frictionless table. The three springs are massless and have the same spring constant k, When one of the springs connecting the two blocks together is at its natural length, the other one is also at its natural length.

1. Draw a figure which clearly shows your choice of generalized coordinates. For each generalized coordinate, determine the associated generalized momentum. Give interpretations of your generalized momenta in terms of simple physical concepts.

2. Find the Hamiltonian equations of motion of the system.

3. Determine the frequencies of the normal modes of oscillation of this system.



THE END!