Homework 1. Due Thursday, January 21

Problem 1. *Rising Snake*
A snake of length L and linear mass density ρ rises from the table. It's head is moving straight up with the constant velocity v. What force does the snake exert on the table?

Problem 2. *1983-Spring-CM-U-1.*
A ball, mass m, hangs by a massless string from the ceiling of a car in a passenger train. At time t the train has velocity $\vec{v}$ and acceleration $\vec{a}$ in the same direction. What is the angle that the string makes with the vertical? Make a sketch which clearly indicates the relative direction of deflection.

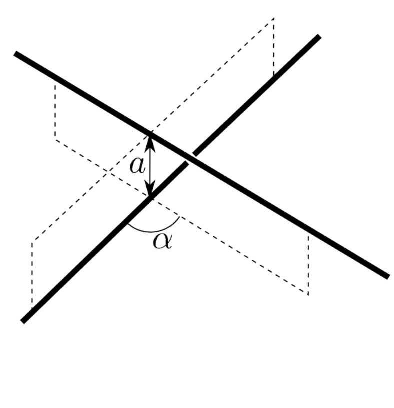
Problem 3. *1984-Fall-CM-U-1.*
Sand drops vertically from a stationary hopper at a rate of 100 gm/sec onto a horizontal conveyor belt moving at a constant velocity, $\vec{v}$, of 10 cm/sec.

1. What force (magnitude and direction relative to the velocity) is required to keep the belt moving at a constant speed of 10 cm/sec?
2. How much work is done by this force in 1.0 second?
3. What is the change in kinetic energy of the conveyor belt in 1.0 second due to the additional sand on it?
4. Should the answers to parts 2. and 3. be the same? Explain.

Problem 4. *1994-Fall-CM-U-1*
Two uniform very long (infinite) rods with identical linear mass density ρ do not intersect. Their directions form an angle α and their shortest separation is a.

1. Find the force of attraction between them due to Newton's law of gravity.
2. Give a dimensional argument to explain why the force is independent of a.
3. If the rods were of a large but finite length L, what dimensional form would the lowest order correction to the force you found in the first part have?

Note: for $A^2 < 1$, $\int_{-\pi/2}^{\pi/2} \frac{d\theta}{1-A^2\sin^2\theta} = \frac{\pi}{\sqrt{1-A^2}}$



Homework 2. Due Thursday, January 28

Problem 1. *1985-Spring-CM-U-3.*
A damped one-dimensional linear oscillator is subjected to a periodic driving force described by a function $F(t)$. The equation of motion of the oscillator is given by

$$m\ddot{x} + b\dot{x} + kx = F(t),$$

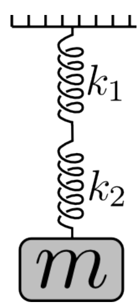
where $F(t)$ is given by

$$F(t) = F_0\left(1 + \sin(\omega t)\right).$$

The driving force frequency is $\omega = \omega_0$ and the damping by $b/2m = \omega_0$, where $\omega_0^2 = k/m$. At $t = 0$ the mass is at rest at the equilibrium position, so that the initial conditions are given by $x(0) = 0$, and $\dot{x}(0) = 0$. Find the solution $x(t)$ for the position of the oscillator vs. time.

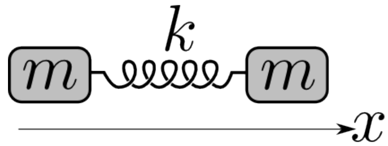
Problem 2. *1986-Spring-CM-U-1.*
A mass m hangs vertically with the force of gravity on it. It is supported in equilibrium by two different springs of spring constants k_1 and k_2 respectively. The springs are to be considered ideal and massless.

Using your own notations (clearly defined) for any coordinates and other physical quantities you need develop in logical steps an expression for the net force on the mass if it is displaced vertically downward a distance y from its equilibrium position. (Clarity and explicit expression of your physical reasoning will be important in the evaluation of your solution to this problem. Your final result should include y, k_1, k_2, and any other defined notations you need.)



Problem 3. *1989-Fall-CM-U-1.*
Two equal masses m are connected by a string of force constant k. They are restricted to motion in the $\hat{x}$ direction.

1. Find the normal mode frequencies.
2. A leftward impulse P is suddenly given to the particle on the right. How long does it take for the spring to reach maximum compression?
3. How far does the mass on the left travel before the spring reaches maximum compression?



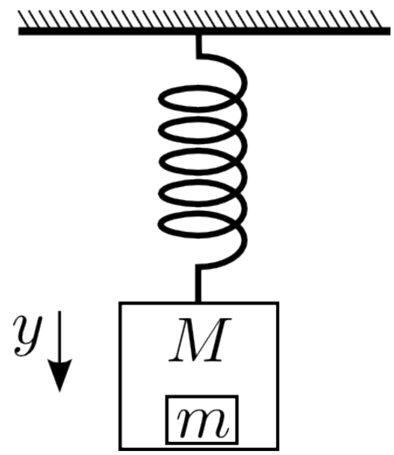
Problem 4. *2001-Spring-CM-G-5*

An ideal massless spring hangs vertically from a fixed horizontal support. A block of mass m rests on the bottom of a box of mass M and this system of masses is hung on the spring and allowed to come to rest in equilibrium under the force of gravity. In this condition of equilibrium the extension of the spring beyond its relaxed length is Δy. The coordinate y as shown in the figure measures the displacement of M and m from equilibrium.

1. Suppose the system of two masses is raised to a position $y = -d$ and released from rest at $t = 0$. Find an expression for $y(t)$ which correctly describes the motion for $t \geq 0$.

2. For the motion described in the previous part, determine an expression for the force of M on m as a function of time.

3. For what value of d is the force on m by M instantaneously zero immediately after m and M are released from rest at $y = -d$?



Homework 3. Due Thursday, February 4

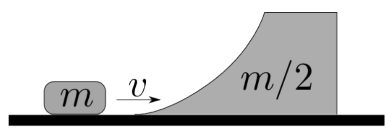
Problem 1. *1984-Fall-CM-U-2.*
Two forces $\mathbf{F}^A$ and $\mathbf{F}^B$ have the following components

$$\mathbf{F}^A : \begin{cases} F_x^A = 6abyz^3 - 20bx^3y^2 \\ F_y^A = 6abxz^3 - 10bx^4y \\ F_z^A = 18abxyz^2 \end{cases} , \qquad \mathbf{F}^B : \begin{cases} F_x^B = 18abyz^3 - 20bx^3y^2 \\ F_y^B = 18abxz^3 - 10bx^4y \\ F_z^B = 6abxyz^2 \end{cases}$$

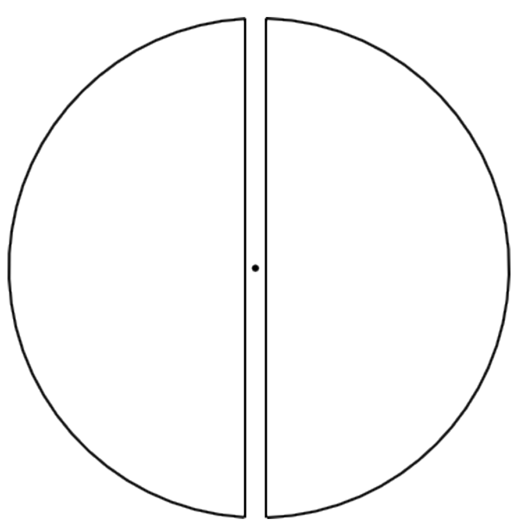
Which one of these is a conservative force? Prove your answer. For the conservative force determine the potential energy function $V(x, y, z)$. Assume $V(0, 0, 0) = 0$.

Problem 2. *1987-Fall-CM-U-1.*
A block of mass m slides on a frictionless table with velocity v. At $x = 0$, it encounters a frictionless ramp of mass $m/2$ which is sitting at rest on the frictionless table. The block slides up the ramp, reaches maximum height, and slides back down.

1. What is the velocity of the block when it reaches its maximum height?
2. How high above the frictionless table does the block rise?
3. What are the final velocities of the block and the ramp?



Problem 3. *1983-Fall-CM-G-5*
Assume that the earth is a sphere, radius R and uniform mass density, ρ. Suppose a shaft were drilled all the way through the center of the earth from the north pole to the south. Suppose now a bullet of mass m is fired from the center of the earth, with velocity v_0 up the shaft. Assuming the bullet goes beyond the earth's surface, calculate how far it will go before it stops.



Problem 4. *1995-Spring-CM-G-2.jpg*

A particle of mass m moves under the influence of a central attractive force

$$F = -\frac{k}{r^2} e^{-r/a}$$

1. Determine the condition on the constant a such that circular motion of a given radius r_0 will be stable.
2. Compute the frequency of small oscillations about such a stable circular motion.

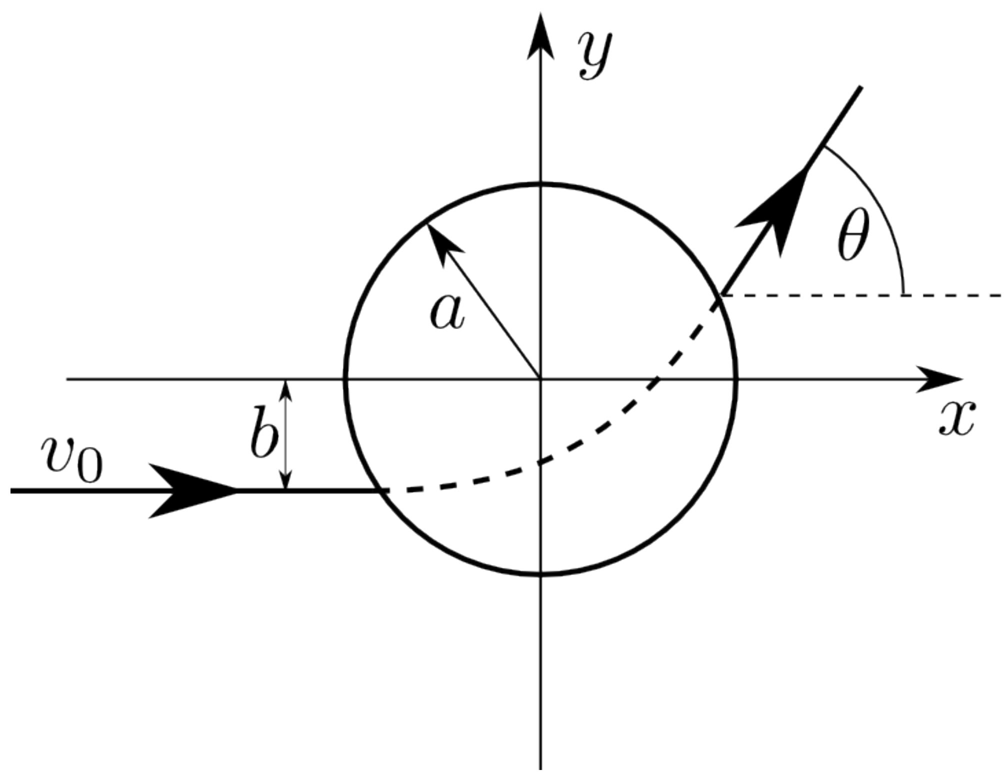
Homework 4. Due Thursday, February 11

Problem 1. *2001-Spring-CM-U-3*
A particle with mass m, confined to a plane, is subject to a force field given by

$$\vec{F}(\vec{r}) = \begin{cases} -k\vec{r}, & \text{for } |r| \leq a \\ 0, & \text{for } |r| > a \end{cases}$$

The particle enters the field from the left horizontally with an initial velocity v_0 and a distance $b < a$ below the force center as shown.

1. Write down the equation of motion for $r \leq a$ in Cartesian coordinates in terms of x, y, and $\omega = \sqrt{k/m}$
2. Give the trajectory of the particle when $r < a$.
3. For $v_0 = a\omega$ find the coordinates of the particle as it exits the region of non-zero force field.
4. For $v_0 = a\omega$, find the deflection angle θ of the departing velocity at the exit point.



Problem 2. *1998-Spring-CM-U-3*

A particle of mass m has velocity v_0 when it is far from the center of a potential of the form:

$$V(r) = \frac{k}{r^4},$$

where k is a positive constant that you can conveniently write as

$$k = \frac{1}{2} m v_0^2 a^4.$$

1. Find r_{min} the distance of the closest approach of the particle to the center of force, as a function of the impact parameter b of the particle.

2. Find an expression for the scattering angle of the particle as a function of the impact parameter b. Your expression may include a definite integral that you need not evaluate, so long as no unknown quantities appear in the integral.

Problem 3. *1994-Fall-CM-G-3.jpg*

A particle of mass m moves under the influence of an attractive central force $F(r) = -k/r^3$, $k > 0$. Far from the center of force, the particle has a kinetic energy E.

1. Find the values of the impact parameter b for which the particle reaches $r = 0$.

2. Assume that the initial conditions are such that the particle misses $r = 0$. Solve for the scattering angle θ_s, as a function of E and the impact parameter b.

Problem 4.

1. Determine the cross-section for a particle of energy E to "fall" to the center of a field $U = -\frac{\alpha}{r^2}$, where $\alpha > 0$.

2. Determine the cross-section for a particle of energy E to "fall" to the center of a field $U = -\frac{\alpha}{r^n}$, where $\alpha > 0$ and $n > 2$.

Homework 5. Due Thursday, February 18

Problem 1. *A string with a tension*
A string of tension T and linear mass density ρ connects two horizontal points distance L apart from each other. y is the vertical coordinate pointing up, and x the is horizontal coordinate.

1. Write down the functional of potential energy of the string vs. the shape of the string $y(x)$. Specify the boundary conditions for the function $y(x)$.

2. Write down the equation which gives the shape of minimal energy for the string.

3. Find the solution of the equation which satisfies the boundary conditions. (Do not try to solve the transcendental equation for one of the constant. Just write it down.)

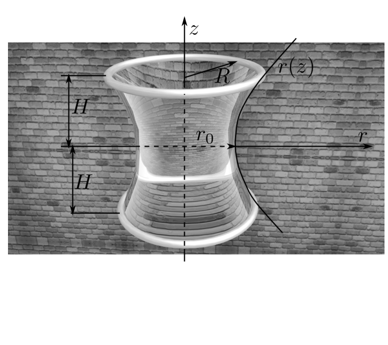
4. In the case $T \gg \rho g L$, the shape is approximately given by $y \approx -\frac{\alpha}{2}x(L-x)$. Find α.

Problem 2. *1995-Spring-CM-G-3*
A soap film is stretched over 2 coaxial circular loops of radius R, separated by a distance $2H$. Surface tension (energy per unit area, or force per unit length) in the film is τ =const. Gravity is neglected.

1. Assuming that the soap film takes en axisymmetric shape, such as illustrated in the figure, find the equation for $r(z)$ of the soap film, with r_0 (shown in the figure) as the only parameter. (Hint: You may use either variational calculus or a simple balance of forces to get a differential equation for $r(z)$).

2. Write a transcendental equation relating r_0, R and H, determine approximately and graphically the maximum ratio $(H/R)_c$, for which a solution of the first part exists. If you find that multiple solutions exist when $H/R < (H/R)_c$, use a good physical argument to pick out the physically acceptable one. (Note: equation $x = \cosh(x)$ has the solution $x \approx \pm 0.83$.)

3. What shape does the soap film assume for $H/R > (H/R)_c$?



Problem 3. *1987-Fall-CM-G-4*

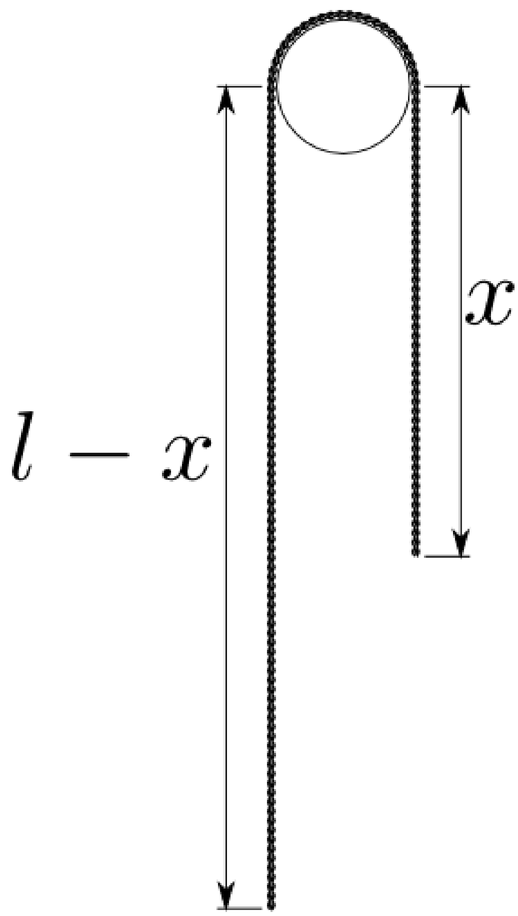
Assume that the sun (mass $M_\odot$) is surrounded by a uniform spherical cloud of dust of density ρ. A planet of mass m moves in an orbit around the sun withing the dust cloud. Neglect collisions between the planet and the dust.

1. What is the angular velocity of the planet when it moves in a circular orbit of radius r?

2. Show that if the mass of the dust within the sphere of the radius r is small compared to $M_\odot$, a nearly circular orbit will precess. Find the angular velocity of the precession.

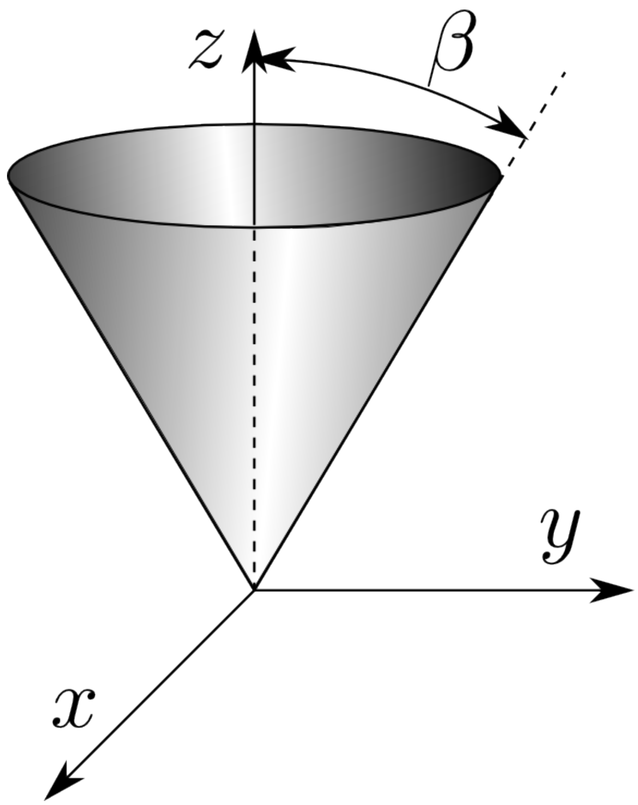
Homework 6. Due Thursday, February 25

Problem 1. *1983-Spring-CM-G-4*
A simple Atwood's machine consists of a heavy rope of length l and linear density ρ hung over a pulley. Neglecting the part of the rope in contact with the pulley, write down the Lagrangian. Determine the equation of motion and solve it. If the initial conditions are $\dot{x} = 0$ and $x = l/2$, does your solution give the expected result?



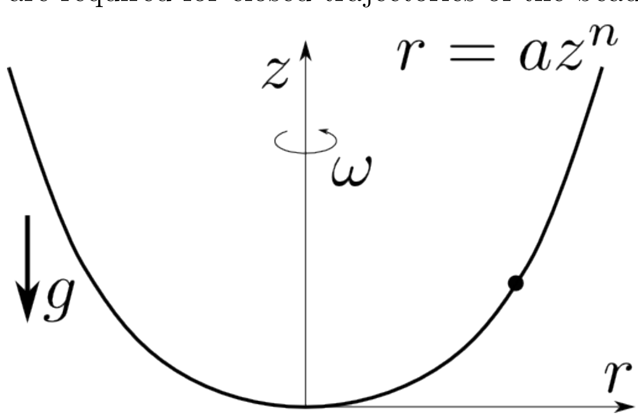
Problem 2. *1993-Spring-CM-G-5.jpg*
Consider a particle of mass m constrained to move on the surface of a cone of half angle β, subject to a gravitational force in the negative z-direction. (See figure.)

1. Construct the Lagrangian in terms of two generalized coordinates and their time derivatives.
2. Calculate the equations of motion for the particle.
3. Show that the Lagrangian is invariant under rotations around the z-axis, and, calculate the corresponding conserved quantity.



Problem 3. *1994-Spring-CM-G-1.jpg*
A bead slides without friction on a stiff wire of shape $r(z) = az^n$, with $z > 0$, $0 < n < 1$, which rotates about the vertical z axis with angular frequency ω, as shown in the figure.

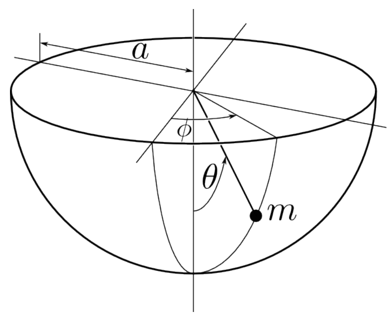
1. Derive the Lagrange equation of motion for the bead.
2. If the bead follows a horizontal circular trajectory, find the height z_0 in terms of n, a, ω, and the gravitational acceleration g.
3. Find the conditions for stability of such circular trajectories.
4. For a trajectory with small oscillations in the vertical direction, find the angular frequency of the oscillations, ω', in terms of n, a, z_0, and ω.
5. What conditions are required for closed trajectories of the bead?



Problem 4. *1996-Fall-CM-G-4*

A particle of mass m slides inside a smooth hemispherical cup under the influence of gravity, as illustrated. The cup has radius a. The particle's angular position is determined by polar angle θ (measured from the negative vertical axis) and azimuthal angle ϕ.

1. Write down the Lagrangian for the particle and identify two conserved quantities.
2. Find a solution where $\theta = \theta_0$ is constant and determine the angular frequency $\dot{\phi} = \omega_0$ for the motion.
3. Now suppose that the particle is disturbed slightly so that $\theta = \theta_0 + \alpha$ and $\dot{\phi} = \omega_0 + \beta$, where α and β are small time-dependent quantities. Obtain, to linear order in α and β the equations of motion for the perturbed motion. Hence find the frequency of the small oscillation in θ that the particle undergoes.



Homework 7. Due Thursday, March 3

Problem 1. *1992-Spring-CM-G-5*
A particle of mass m is moving on a sphere of radius a, in the presence of a velocity dependent potential $U = \sum_{i=1,2} \dot{q}_i A_i$, where $q_1 = \theta$ and $q_2 = \phi$ are the generalized coordinates of the particle and $A_1 \equiv A_\theta$, $A_2 \equiv A_\phi$ are given functions of θ and ϕ.

1. Calculate the generalized force defined by
$$Q_i = \frac{d}{dt}\frac{\partial U}{\partial \dot{q}_i} - \frac{\partial U}{\partial q_i}.$$
2. Write down the Lagrangian and derive the equation of motion in terms of θ and ϕ.
3. For $A_\theta = 0$, $A_\phi = g(1 - \cos\theta)$, where g is a constant, describe the symmetry of the Lagrangian and find the corresponding conserved quantity.
4. In terms of three dimensional Cartesian coordinates, *i.e.*, $q_i = x_i$ show that Q_i can be written as $\vec{Q} = \vec{v} \times \vec{B}$, where $v_i = \dot{x}_i$. Find $\vec{B}$ in terms of $\vec{A}$.

Problem 2. *1999-Spring-CM-G-4.jpg*
A particle of mass m is observed to move in a central field following a planar orbit (in the $x - y$ plane) given by.
$$r = r_0 e^{-\theta},$$
where r and θ are coordinates of the particle in a polar coordinate system.

1. Prove that, at any instant in time, the particle trajectory is at an angle of 45° to the radial vector.
2. When the particle is at $r = r_0$ it is seen to have an angular velocity $\Omega > 0$. Find the total energy of the particle and the potential energy function $V(r)$, assuming that $V \to 0$ as $r \to +\infty$.
3. Determine how long it will take the particle to spiral in from $r = r_0$, to $r = 0$.

Problem 3. *Noether's theorem*
Which components (or their combinations) of momentum $\vec{P}$ and angular momentum $\vec{M}$ are conserved in motion in the following fields.

1. the field of an infinite homogeneous plane,
2. that of an infinite homogeneous cylinder,
3. that of an infinite homogeneous prism,
4. that of two points,

5. that of an infinite homogeneous half plane,
6. that of a homogeneous cone,
7. that of a homogeneous circular torus,
8. that of an infinite homogeneous cylindrical helix of pitch h.

Homework 8. Due Thursday, March 10

Problem 1. *1985-Spring-CM-G-5*
A bead slides without friction on a wire in the shape of a cycloid:

$$x = a(\theta - \sin\theta)$$
$$y = a(1 + \cos\theta)$$

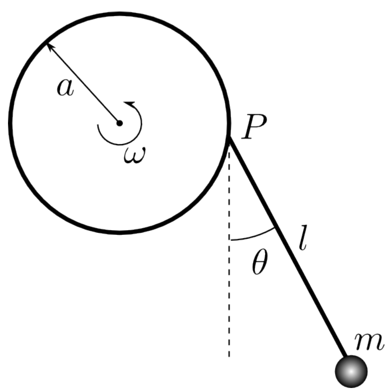
1. Write down the Hamiltonian of the system.

2. Derive Hamiltonian's equations of motion.

Problem 2. *1991-Fall-CM-G-5*
A simple pendulum of length l and mass m is suspended from a point P that rotates with constant angular velocity ω along the circumference of a vertical circle of radius a.

1. Find the Hamiitionian function and the Hamiltonian equation of motion for this system using the angle θ as the generalized coordinate.

2. Do the canonical momentum conjugate to θ and the Hamiltonian function in this case correspond to physical quantities? If so, what are they?



Problem 3. *1991-Spring-CM-G-5*
A particle is constrained to move on a cylindrically symmetric surface of the form $z = (x^2 + y^2)/(2a)$. The gravitational force acts in the $-z$ direction.

1. Use generalized coordinates with cylindrical symmetry to incorporate the constraint and derive the Lagrangian for this system.

2. Derive the Hamiltonian function, Hamilton's equation, and identify any conserved quantity and first integral of motion.

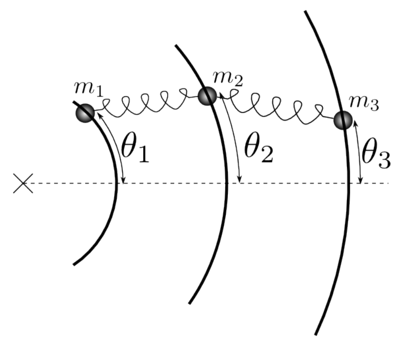
3. Find the radius r_0 of a steady state motion in r having angular momentum l.

4. Find the frequency of small radial oscillations about this steady state.

Homework 9. Due Thursday, March 17

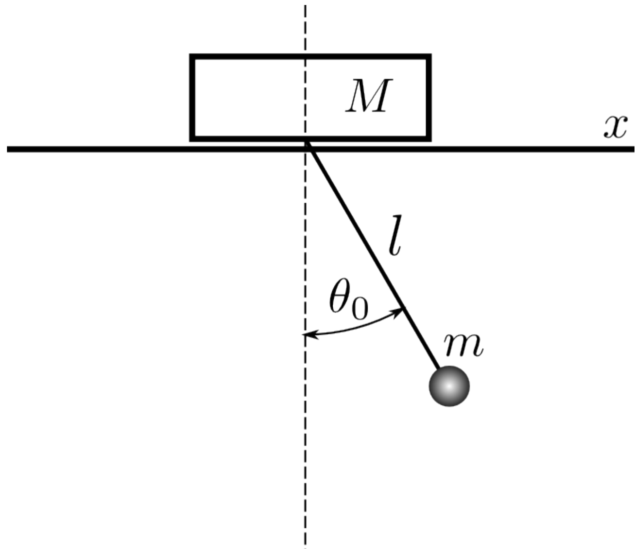
Problem 1. *1991-Spring-CM-G-4*
Three particles of masses $m_1 = m_0$, $m_2 = m_0$, and $m_3 = m_0/3$ are restricted to move in circles of radius a, $2a$, and $3a$ respectively. Two ideal springs of natural length much smaller than a and force constant k link particles 1, 2 and particles 2, 3 as shown.

1. Determine the Lagrangian of this system in terms of polar angles θ_1, θ_2, θ_3 and parameters m_0, a, and k.
2. For small oscillations about an equilibrium position, determine the system's normal mode frequencies in term of $\omega_0 = \sqrt{k/m_0}$.
3. Determine the normalized eigenvector corresponding to each normal mode and describe their motion physically.
4. What will happen if the natural length of the springs is a?



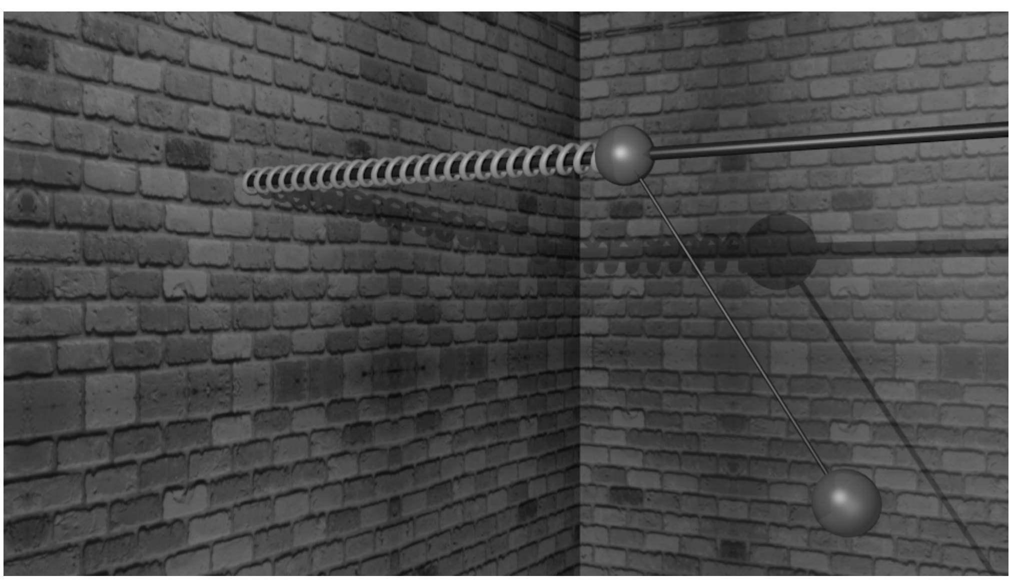
Problem 2. *1994-Spring-CM-G-3.jpg*
A simple pendulum of length l and mass m is attached to s block of mass M, which is free to slide without friction in a horizontal direction. All motion is confined to a plane. The pendulum is displaced by a small angle θ_0 and released.

1. Choose a convenient set of generalized coordinates and obtain Lagrange's equations of motion. What are the constants of motion?
2. Make the small angle approximation ($\sin\theta \approx \theta$, $\cos\theta \approx 1$) and solve the equations of motion. What is the frequency of oscillation of the pendulum, and what is the magnitude of the maximum displacement of the block from its initial position?



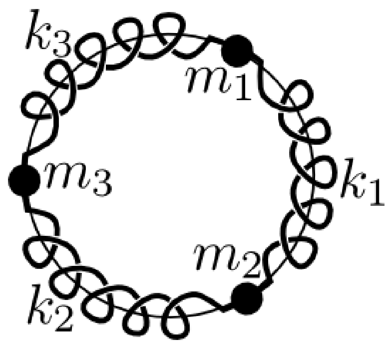
Problem 3. *1995-Fall-CM-G-1.jpg*
Consider a system of two point-like weights, each of mass M, connected by a massless rigid rod of length l . The upper weight slides on a horizontal frictionless rail and is connected to a horizontal spring, with spring constant k, whose other end is fixed to a wall as shown below. The lower weight swings on the rod, attached to the upper weight and its motion is confined to the vertical plane.

1. Find the exact equations of motion of the system.
2. Find the frequencies of small amplitude oscillation of the system.
3. Describe qualitatively the modes of small oscillations associated with the frequencies you found in the previous part.



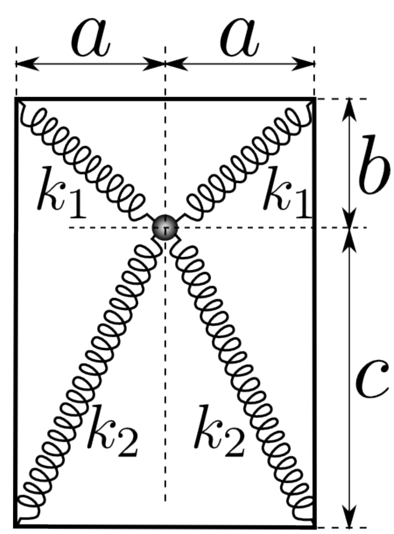
Homework 10. Due Thursday, March 31

Problem 1. *1985-Fall-CM-U-3.*
Three particles of the same mass $m_1 = m_2 = m_3 = m$ are constrained to move in a common circular path. They are connected by three identical springs of stiffness $k_1 = k_2 = k_3 = k$, as shown. Find the normal frequencies and normal modes of the system.



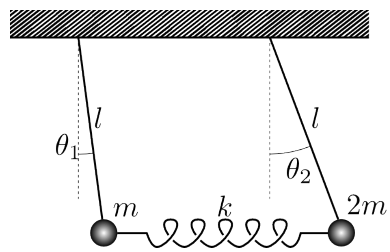
Problem 2. *1991-Fall-CM-U-2.*
A steel ball of mass M is attached by massless springs to four corners of a $2a$ by $b+c$ horizontal, rectangular frame. The springs constants k_1 and k_2 with corresponding equilibrium lengths $L_1 = \sqrt{a^2 + b^2}$ and $L_2 = \sqrt{a^2 + c^2}$ as shown. Neglecting the force of gravity,

1. Find the frequencies of small amplitude oscillations of the steel ball in the plane of rectangular frame.
2. Identify the type of motion associated with each frequency.
3. Is the oscillation of the steel ball perpendicular to the plane of the rectangular frame harmonic? Why or why not?



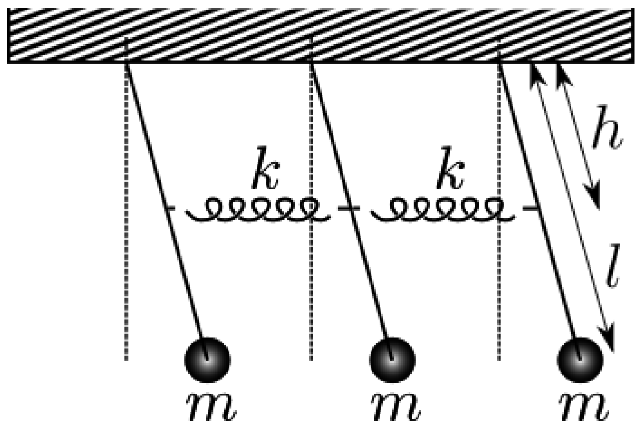
Problem 3. *1983-Spring-CM-G-6*

Two pendula made with massless strings of length l and masses m and $2m$ respectively are hung from the ceiling. The two masses are also connected by a massless spring with spring constant k. When the pendula are vertical the spring is relaxed. What are the frequencies for small oscillations about the equilibrium position? Determine the eigenvectors. How should you initially displace the pendula so that when they are released, only one eigen frequency is excited. Make the sketches to specify these initial positions for both eigen frequencies.

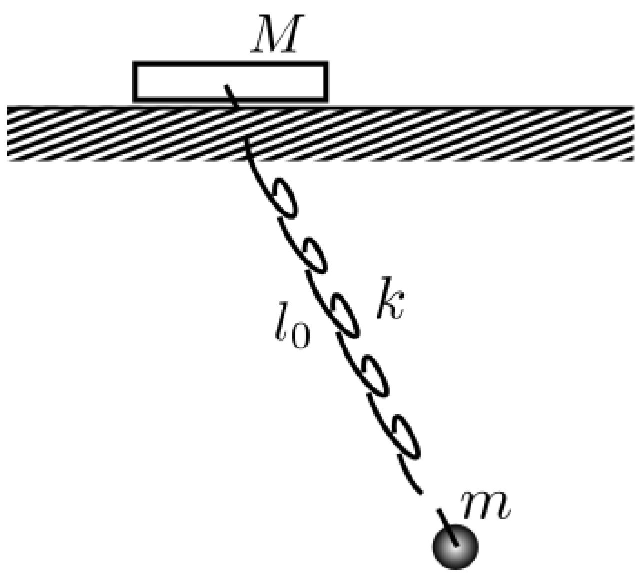


Homework 11. Due Thursday, April 7

Problem 1. *Three pendulums*
Three identical pendulums are connected by the springs as shown on the figure. Find the normal modes



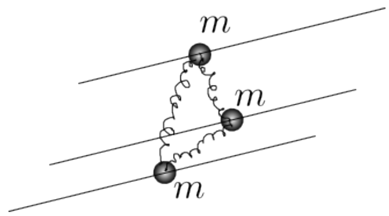
Problem 2. *A pendulum a spring and a block*
A ball of mass m is hanging on a weightless spring of spring constant k, which is attached to a block of mass M. The block can move along the table without friction. The length of the unstretched spring is l_0. Find the normal modes of small oscillations.



Problem 3.
Three infinite horizontal rails are at distance l from each other. Three beads of equal masses m can slide along the rails without friction. The three beads are connected by identical springs of constants k and equilibrium/unstretched length $l/2$.

1. Find the normal modes and the normal frequencies.

2. At time $t = 0$ one of the masses is suddenly given a velocity v. What will be the velocity of the whole system of three masses (average velocity) in the long time?

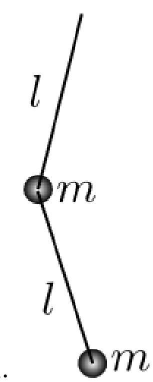


Homework 12. Due Thursday, April 14

Problem 1.
Determine the normal/eigen frequencies for the four equal masses m on a ring. The masses are connected by the identical springs of spring constant k along the ring.

Problem 2.
Determine the normal/eigen frequencies for two equal masses m on a ring of radius R. The masses are connected by two identical springs of spring constant k along the ring. The ring is in the vertical plane. The acceleration of the free fall is g. Consider only the case of hard springs: $\frac{mg}{kR} \ll 1$.

Problem 3.



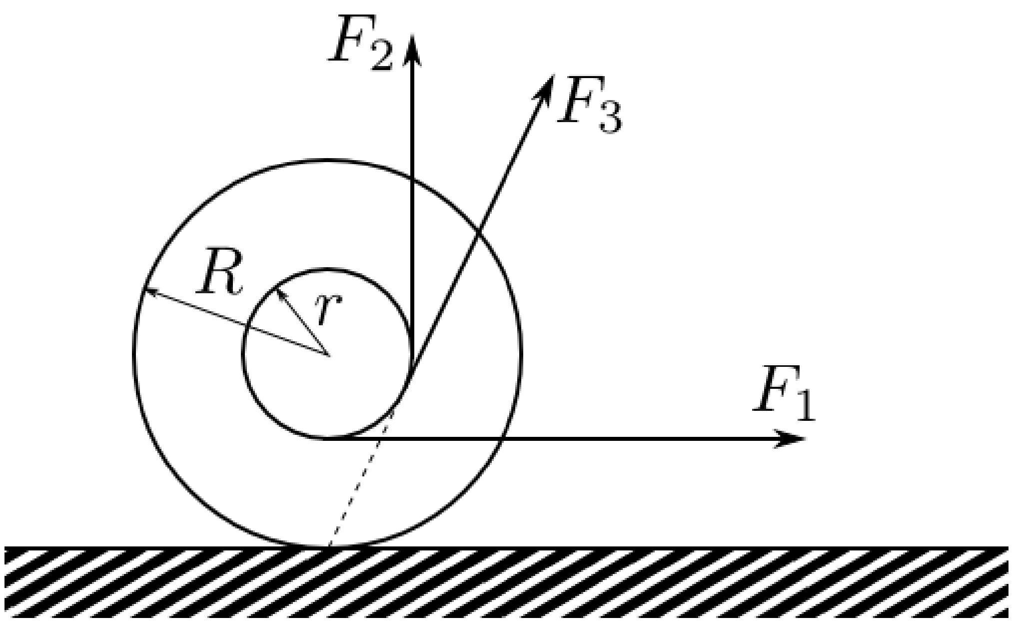
Find normal frequencies and normal modes of a double pendulum.

Homework 13. Due Thursday, April 21

Problem 1. *1983-Fall-CM-G-4*
A yo-yo (inner radius r, outer radius R) is resting on a horizontal table and is free to roll. The string is pulled with a constant force F. Calculate the horizontal acceleration and indicate its direction for three different choices of F. Assume the yo-yo maintains contact with the table and can roll but does not slip.

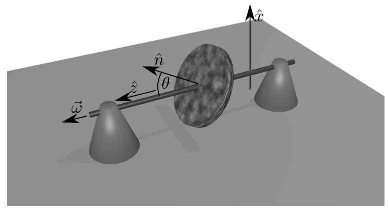
1. $F = F_1$ is horizontal,
2. $F = F_2$ is vertical,
3. $F = F_3$ (its line of action passes through the point of contact of the yo-yo and table.)

Approximate the moment of inertia of the yo-yo about its symmetry axis by $I = \frac{1}{2}MR^2$ here M is the mass of the yo-yo.



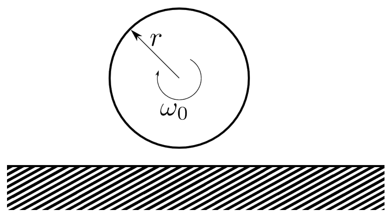
Problem 2. *1985-Fall-CM-G-6*
A disk is rigidly attached to an axle passing through its center so that the disc's symmetry axis $\hat{n}$ makes an angle θ with the axle. The moments of inertia of the disc relative to its center are C about the symmetry axis $\hat{n}$ and A about any direction $\hat{n}'$ perpendicular to $\hat{n}$. The axle spins with constant angular velocity $\vec{\omega} = \omega\hat{z}$ ($\hat{z}$ is a unit vector along the axle.) At time $t = 0$, the disk is oriented such that the symmetry axis lies in the $X - Z$ plane as shown.

1. What is the angular momentum, $\vec{L}(t)$, expressed in the space-fixed frame.
2. Find the torque, $\vec{\tau}(t)$, which must be exerted on the axle by the bearings which support it. Specify the components of $\vec{\tau}(t)$ along the space-fixed axes.



Problem 3. *1987-Fall-CM-G-6*

A uniform solid cylinder of radius r and mass m is given an initial angular velocity ω_0 and then dropped on a flat horizontal surface. The coefficient of kinetic friction between the surface and the cylinder is μ. Initially the cylinder slips, but after a time t pure rolling without slipping begins. Find t and v_f, where v_f is the velocity of the center of mass at time t.

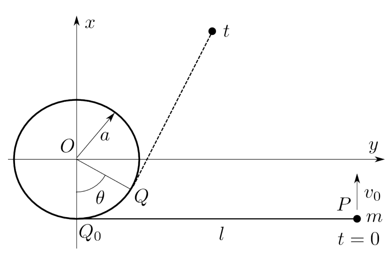


Homework 14. Due Thursday, April 28

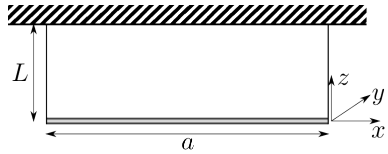
Problem 1. *1998-Spring-CM-G-4.jpg*
A mass m moves on a smooth, frictionless horizontal table. It is attached by a massless string of constant length $l = 2\pi a$ to a point Q_0 of an immobile cylinder. At time $t_0 = 0$ the mass at point P is given an initial velocity v_0 at right angle to the extended string, so that it wraps around the cylinder. At a later time t, the mass has moved so that the contact point Q with the cylinder has moved through an angle θ, as shown. The mass finally reaches point Q_0 at time t_f.

1. Is kinetic energy constant? Why or why not?
2. Is the angular momentum about O, the center of the cylinder, conserved? Why or why not?
3. Calculate as a function of θ, the speed of the contact point Q, as it moves around the cylinder. Then calculate the time it takes mass m to move from point P to point Q_0.
4. Calculate the tension T in the string as a function of m, v, θ, and a.
5. By integrating the torque due to T about O over the time it lakes mass m to move from point P to point Q_0, show that the mass's initial angular momentum mv_0l is reduced to zero when the mass reaches point Q_0. Hint: evaluate
$$\int_0^{t_f} \Gamma dt = \int_0^{2\pi} \frac{\Gamma}{d\theta/dt} d\theta.$$
6. What is the velocity (direction and magnitude) of m when it hits Q_0?
7. What is the tension T when the mass hits Q_0?

You may wish to use the (x, y) coordinate system shown.



Problem 2. *2001-Fall-CM-G-4.jpg*
A rigid rod of length a and mass m is suspended by equal massless threads of length L fastened to its ends. While hanging at rest, it receives a small impulse $\vec{J} = J_0\hat{y}$ at one end, in a direction perpendicular to the axis of the rod and to the thread. It then undergoes a small oscillation in the $x - y$ plane. Calculate the normal frequencies and the amplitudes of the associated normal modes in the subsequent motion.



Problem 3. *2001-Spring-CM-G-4.jpg*
A rotor consists of two square flat masses: m and $2m$ as indicated. These masses are glued so as to be perpendicular to each other and rotated about a an axis bisecting their common edge such that $\vec{\omega}$ points in the $x - z$ plane 45° from each axis. Assume there is no gravity.

1. Find the principal moments of inertia for this rotor, I_{xx}, I_{yy}, and I_{zz}. Note that off-diagonal elements vanish, so that x, y, and z are principal axes.
2. Find the angular momentum, $\vec{L}$ and its direction.
3. What torque vector $\vec{\tau}$ is needed to keep this rotation axis fixed in time?

Give all vectors in components of the internal $x - y - z$ system of coordinates.