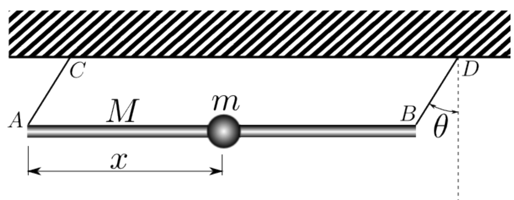
EXAM 1. Thursday, March 24, 2016, 5:30-6:45pm

Problem 1. *1993-Fall-CM-G-3.jpg*
A uniform smooth rod AB, of mass M hangs from two fixed supports C and D by light inextensible strings AC and BD each of length l, as shown in the figure. The rod is horizontal and $AB = CD = L \gg l$. A bead of mass m is located at the center of the rod and can slide freely on the rod. Let θ be the inclination of the strings to the vertical, and let x be the distance of the bead from the end of the rod (A). The initial condition is $\theta = \alpha < \pi/2$, $\dot{\theta} = 0$, $x = L/2$, and $\dot{x} = 0$. Assume the system moves in the plane of the figure.

1. Obtain the Lagrangian $\mathcal{L} = \mathcal{L}(\theta, \dot{\theta}, x, \dot{x})$ and write down the Lagrange's equations of motion for x and θ.

2. Obtain the first integrals of the Lagrange's equations of the motion for x and θ subject to the initial condition.

3. Find the speeds of the bead and the rod at $\theta = 0$.



Problem 2. *1984-Spring-CM-U-2.*
A particle of mass m moves subject to a central force whose potential is $V(r) = Kr^3$, $K > 0$.

1. For what kinetic energy and angular momentum will the orbit be a circle of radius a about the origin?

2. What is the period of this circular motion?

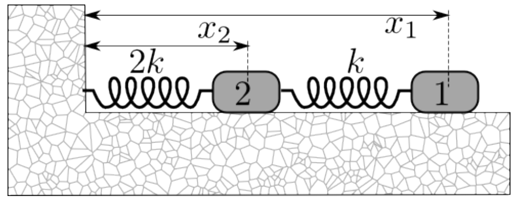
3. If the motion is slightly disturbed from this circular orbit, what will be the period of small radial oscillations about $r = a$?

Problem 3. *1989-Spring-CM-U-3.*
A coupled oscillator system is constructed as shown, $m_1 = m$, and $m_2 = 2m$. Assume that the two springs are massless, and that the motion of the system is only in one dimension with no damping.

1. Find the eigenfrequencies and eigenvectors of the system.

2. Let L_1 and L_2 be the equilibrium positions of masses 1 and 2, respectively. Find the solution for all times $t \geq 0$ for $x_1(t)$ and $x_2(t)$ for the initial conditions:

$$x_1(t=0) = L_1; \qquad dx_1/dt = -V_0 \quad \text{at } t = 0$$
$$x_2(t=0) = L_2; \qquad dx_2/dt = 0 \quad \text{at } t = 0.$$

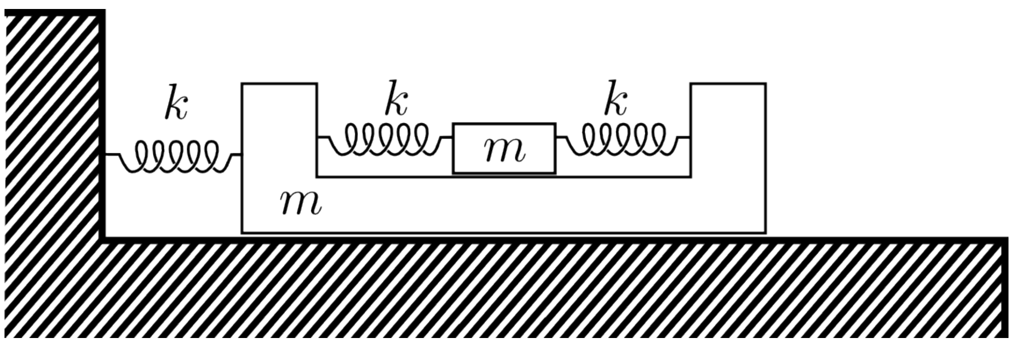


x_1
x_2
$2k$
k
2
1

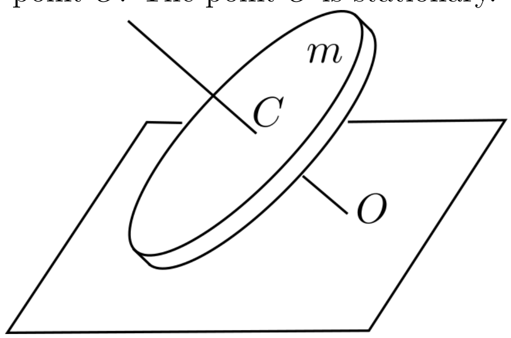
EXAM 2. Final. Tuesday, May 10, 2015, 3:30-5:30pm

Problem 1. *1998-Fall-CM-U-3*
The two "blocks" in the figure below are frictionless and have the same mass m. The larger block is sitting on a horizontal, frictionless table. The three springs are massless and have the same spring constant k, When one of the springs connecting the two blocks together is at its natural length, the other one is also at its natural length.

1. Draw a figure which clearly shows your choice of generalized coordinates. For each generalized coordinate, determine the associated generalized momentum. Give interpretations of your generalized momenta in terms of simple physical concepts.
2. Find the Hamiltonian equations of motion of the system.
3. Determine the frequencies of the normal modes of oscillation of this system.



Problem 2. *A coin*
A uniform thin disc of mass m rolls without slipping on a horizontal plane. The disc makes an angle α with the plane, the center of the disc C moves around the horizontal circle with a constant speed v. The axis of symmetry of the disc CO intersects the horizontal plane in the point O. The point O is stationary. Find the kinetic energy of the disc.



Problem 3. *1984-Fall-CM-G-5*
A ladder of length L and mass M rests against a smooth wall and slides without friction on wall and floor. Initially the ladder is at rest at an angle $\alpha = \alpha_0$ with the floor. (For the ladder the moment of inertia about an axis perpendicular to and through the center of the ladder is $\frac{1}{12}ML^2$).

1. Write down the Lagrangian and Lagrange equations for $\alpha(t)$.
2. Find the first integral of motion in the angle α.
3. Determine the force exerted by the wall on the ladder as a function of α.
4. Determine the angle α_c at which the ladder leaves the wall.

THE END!