EXAM 1. Tuesday, October 11, 2017

Problem 1. *Name. 1pt.*
Write down your name. Clearly. In block letters!

Problem 2. *1987-Fall-SM-G-4*
Consider a cyclic engine operating with one mole of an ideal monoatomic gas in between two baths with temperatures T_a and $2T_a$ in the cycle $a \to b \to c \to d \to a$. V_a and T_a are the volume and the temperature of the gas in point a.

$a \to b$ is isobaric increase of temperature from T_a to $2T_a$

$b \to c$ isothermal expansion to the volume $3V_a$

$c \to d$ decrease of temperature back to T_a at constant volume

$d \to a$ isothermal compression back to the volume V_a

All processes are reversible.

a. Calculate the efficiency of this engine and compare it to the maximum possible efficiency for an engine operating between T_a and $2T_a$.

b. What is the net entropy change of the gas in one cycle?

c. What is the net change of the energy of the gas in one cycle?

d. What is the net change of the entropy of the hot thermal bath during one cycle, if all processes are reversible?

e. What is the net change of the entropy of the cold thermal bath during one cycle, if all processes are reversible?

Problem 3. *1998-Spring-SM-G-4*
Consider a soap film supported by a wire frame of fixed length l along one direction and of varying length x along the other direction. Because of surface tension σ, there is a force $2\sigma l$ tending to contract the film. Take $\sigma(T, x) = \sigma_0 - \alpha T$, where σ_0 and α are independent of T and x.

1. In one sentence, explain why the force is $2\sigma l$, rather than σl.
2. Express the energy change dE of the film due to work δW associated with the surface tension and heat δQ absorbed by the film through the atmosphere.
3. Calculate the work W done on the film when it is stretched at a constant temperature T_0 from length 0 to x.
4. Calculate the entropy change of the film when it is stretched at constant temperature T_0 from length 0 to x.

5. Calculate the change in energy $\Delta E = E(x) - E(0)$ when the film is stretched at constant temperature T_0 from length 0 to x.

Problem 4. *1984-Fall-SM-U-1*

A cylinder closed at both ends equipped with insulating (adiabatic) walls, and is divided into two parts with a frictionless, insulating, movable piston. The gases on both sides are initially at equilibrium with identical pressure, volume, and temperature (P_0, V_0, T_0). The gas is ideal with $C_V = 3R/2$ and $C_P/C_V = 5/3$. By means of a heating coil in the gas on the left hand side, heat is slowly supplied to the gas on the left until the pressure reaches $32P_0$. In terms of P_0, V_0, and T_0

1. What is the final right hand volume?
2. What is the final right hand temperature?
3. What is the final left hand temperature?
4. How much heat must be supplied to the gas on the left?
5. How much work is done on the gas on the right?
6. What is entropy change of the gas on the right?
7. Compute the entropy change of the gas on the left.

EXAM 2. Final. Saturday, December 10, 2016, 12:00-4:00pm

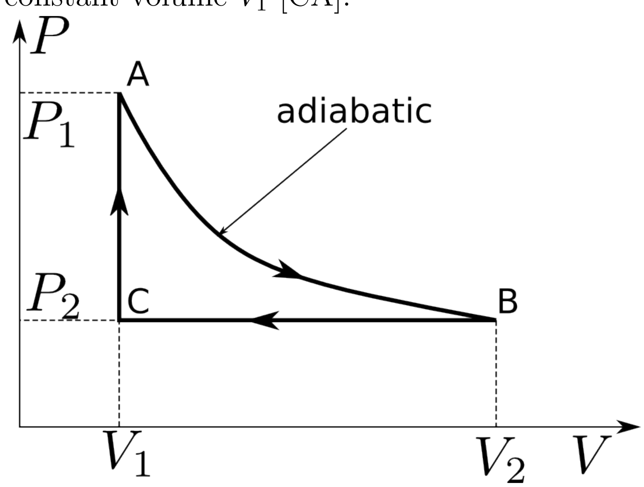
Write down your name. Clearly. In block letters!

Problem 1. *1992-Spring-SM-U-1*
An ensemble of non-interacting pairs of Ising spins is in a magnetic field h and at temperature T. Each spin variable s_i^z can only take on values $s^z = \pm 1$. The two spins within each pair interact according to the Hamiltonian

$$H = -J s_1^z s_2^z - \mu_B h \left(s_1^z + s_2^z\right), \qquad \text{with } J > 0$$

1. Enumerate the possible states of a single pair and compute their corresponding energies.
2. Derive an expression for the average value of a spin, $\langle s_i^z \rangle$, $(i = 1, 2)$ as a function of J, T and h.
3. Given the above model, determine whether there exists a temperature T_c for which $\langle s_i^z \rangle$ can be non-zero at $h = 0$. Evaluate T_c.

Problem 2. *1995-Fall-SM-U-2*
An ideal gas is expanded adiabatically from (P_1, V_1) to (P_2, V_2) [AB in figure below]. Then it is compressed at constant pressure to (P_2, V_1) [BC]. Finally the pressure is increased to P_1 at constant volume V_1 [CA].



1. Calculate W_{BC}, the work done by gas in going from B to C.
2. Calculate W_{CA}, the work done by gas in going from C to A.
3. For an ideal gas, show that $C_P = C_V + Nk$, and hence that $C_V = Nk/(\gamma - 1)$, where $\gamma \equiv C_P/C_V$. (Here N is the number of molecules and k is the Boltzmann constant.)
4. Calculate W_{AB} the work done by gas in going from A to B, in terms of γ, P_2, V_2, P_1, and V_1.

5. Calculate Q_{CA}, the heat absorbed by gas in going from C to A, in terms of γ, V_1, V_2, P_2 and P_1.

6. Calculate the efficiency $\eta \equiv W/Q_{CA}$ of the engine, and show that it is given by

$$\eta = 1 - \gamma \frac{1 - V_2/V_1}{1 - P_1/P_2}$$

(Why do you need to divide by Q_{CA} and not by $Q_{CA} + Q_{BC}$?)

Problem 3. *1995-Spring-SM-U-2*
Consider a one-dimensional (non-harmonic) oscillator with energy given by

$$E = \frac{p^2}{2m} + bx^4,$$

where p is the momentum and b is some constant. Suppose this oscillator is in thermal equilibrium with a heat bath at a sufficiently high temperature T so that classical mechanics is valid.

1. Compute its mean kinetic energy as a fraction of kT.

2. Compute its mean potential energy as a fraction of kT.

3. Consider a collection of such non-interacting oscillators all at thermal equilibrium in one-dimension. What is the specific heat (per particle) of this system?

[**Hint:** You might use

$$\int_0^\infty x^{n-1} e^{-x} dx = \Gamma(n), \qquad n \neq -1, -2, \ldots$$

or an integration by parts in solving this problem.]

Problem 4. *1990-Fall-SM-G-4*
Let $\epsilon_{\vec{p}} = \rho(|p_x| + |p_y| + |p_z|)$ be the energy-momentum relation of a conducting electrons in a certain (fictitious) metal, where ρ is a constant with the dimensions of velocity, and $|x|$ denotes the absolute value of x.

1. Draw a picture of a typical constant-energy surface $\epsilon_{\vec{p}} = \epsilon$, for the above dispersion $\epsilon_{\vec{p}}$.

2. Express the Fermi energy ϵ_F as a function of the electron density n for this metal. (Note, that electrons have spin 1/2.)

3. Calculate the electronic density of states (per unit volume) as a function of ϵ for this metal. Denote it as $D(\epsilon)$.

4. Calculate the total energy of N electrons at $T = 0$ in this metal. Express E_0/N in terms of ϵ_F.

5. Now let the temperature T be finite. Express N and the total energy E as functions of T and the chemical potential μ. The expressions can involve integrals which you do not have to evaluate.

THE END!