Homework 1. Due Tuesday, January 23, 12:45pm

Problem 1. *Problem 5.13 from Classical Mechanics by J. R. Tylor*
The potential energy of a one-dimensional mass m at a distance r from origin is

$$U(r) = U_0 \left(\frac{r}{R} + \lambda^2 \frac{R}{r} \right)$$

for $0 < r < \infty$, with U_0, R, and λ all positive constants.

1. Find the equilibrium position r_0.
2. Let x be the distance from equilibrium and show that, for small x, the potential energy has the form $U = \text{const.} + \frac{1}{2}kx^2$.
3. What is the angular frequency of the oscillations in terms of U_0, R, and λ (and, of course, m)?

Problem 2. *Problems 7.30 from Classical Mechanics by J. R. Tylor*
Consider a pendulum of length l suspended inside a railroad car that is being forced to accelerate with a constant acceleration a.

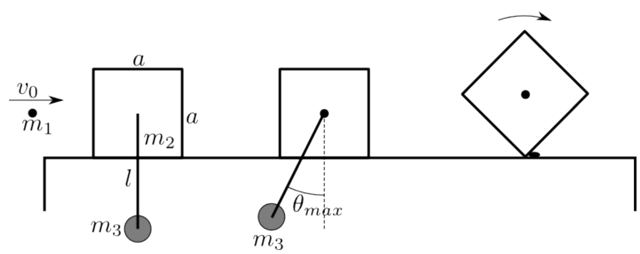
1. Write down the Lagrangian for the system and the equation of motion for the angle ϕ — the angle between the pendulum and the vertical.
2. Find the equilibrium angle ϕ_0.
3. Show, that the equilibrium is stable.
4. Find the angular frequency of the small oscillations around the equilibrium.

Homework 2. Due Tuesday, January 30, 12:45pm

Problem 1. *1994-Spring-CM-U-1*
A bullet of mass m_1, is fired with velocity v_0 at a solid cube of side a on the frictionless table. The cube has mass of m_2 and supports a pendulum of mass m_3 and length l. The cube and pendulum are initially at rest. The bullet becomes embedded in the center of the cube instantaneously after the collision.

1. If θ_{max} is the maximum angle through which the pendulum swings, find the velocity v_0 of the incident bullet.

2. When the pendulum's swing reaches the maximum angle, the pendulum's string is cut off. Therefore the solid cube slides and hits a small obstacle which stops the leading edge of the cube, forcing it to begin rotating about the edge. Find the minimum value of v_0 such that the cube will flip over. Note that the moment of inertia of the cube about an axis along one of its edge is $\frac{2}{3}Ma^2$. Assume the bullet is a point located at the center of the cube.



Problem 2. *2001-Spring-CM-U-3*
A particle with mass m, confined to a plane, is subject to a force field given by

$$\vec{F}(\vec{r}) = \begin{cases} -k\vec{r}, & \text{for } |r| \le a \\ 0, & \text{for } |r| > a \end{cases}$$

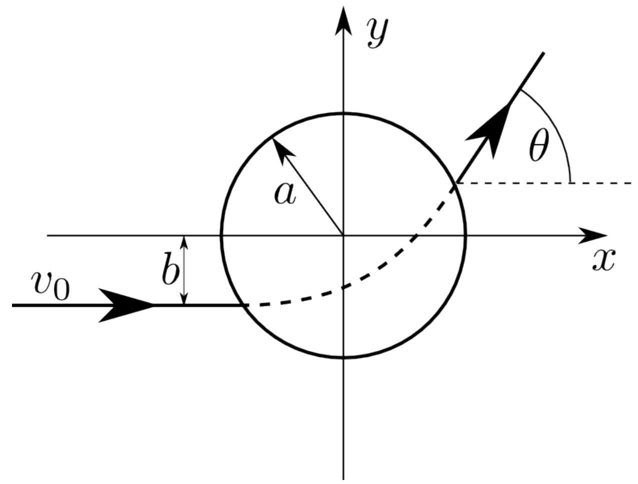
The particle enters the field from the left horizontally with an initial velocity v_0 and a distance $b < a$ below the force center as shown.

1. Write down the equation of motion for $r \le a$ in Cartesian coordinates in terms of x, y, and $\omega = \sqrt{k/m}$

2. Give the trajectory of the particle when $r < a$.

3. For $v_0 = a\omega$ find the coordinates of the particle as it exits the region of non-zero force field.

4. For $v_0 = a\omega$, find the deflection angle θ of the departing velocity at the exit point.



y
a
θ
b
v_0
x

Homework 3. Due Tuesday, February 6, 12:45pm

Problem 1.
A random process has a probability density for a single measurable x given by

$$\rho(x) = Ae^{-\frac{(x-x_0)^2}{2\sigma^2}}$$

where x_0 and σ are known numbers.

1. Find the constant A.
2. Find the average value of x, $\langle x \rangle$.
3. Find the standard deviation $\sqrt{\langle (x - \langle x \rangle)^2 \rangle}$ (be careful with the order of operations).

Problem 2.
A particle in a beam has velocity V and internal energy ϵ. The particles randomly disintegrate inside the detector into two particles of mass m_1 and m_2. The detector measures the kinetic energy of a particles of mass m_1 irrespective of the direction of their velocities.

1. What is the average kinetic energy measured by the detector?.
2. Find the standard deviation of the kinetic energy measured by the detector?

Problem 3.
A particle in a beam has velocity V and internal energy ϵ which is less than the kinetic energy of the particles in the beam. The particles randomly disintegrate inside the detector into two particles A and B of equal mass m. The detector measures the angle θ_L the velocity of the particle A makes with the velocity V of the beam.

Find the average of the tangent of the angle measured by the detector. Express the answer in therms of the speed of the particle A in the center-of-mass frame and the velocity V.

Homework 4. Due Tuesday, February 13, 12:45pm

Problem 1. *1989-Spring-CM-U-1.*
A particle of mass m scatters off a second particle with mass M according to a potential

$$U(r) = \frac{\alpha}{r^2}, \qquad \alpha > 0$$

Initially m has a velocity v_0 and approaches M with an impact parameter b. Assume $m \ll M$, so that M can be considered to remain at rest during the collision.

1. Find the distance of closest approach of m to M.

2. Find the laboratory scattering angle. (Remember that M remains at rest.)

Problem 2. *1998-Spring-CM-U-3*
A particle of mass m has velocity v_0 when it is far from the center of a potential of the form:

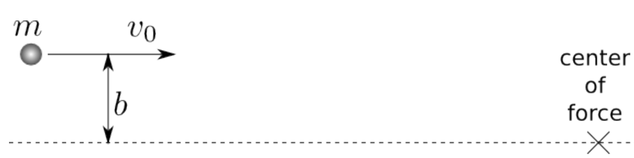
$$V(r) = \frac{k}{r^4},$$

where k is a positive constant that you can conveniently write as

$$k = \frac{1}{2} m v_0^2 a^4.$$

1. Find r_{min} the distance of the closest approach of the particle to the center of force, as a function of the impact parameter b of the particle.

2. Find an expression for the scattering angle of the particle as a function of the impact parameter b. Your expression may include a definite integral that you need not evaluate, so long as no unknown quantities appear in the integral.



For Honors.

Problem 3. *1994-Fall-CM-G-3.jpg*
A particle of mass m moves under the influence of an attractive central force $F(r) = -k/r^3$, $k > 0$. Far from the center of force, the particle has a kinetic energy E.

1. Find the values of the impact parameter b for which the particle reaches $r = 0$.

2. Assume that the initial conditions are such that the particle misses $r = 0$. Solve for the scattering angle θ_s, as a function of E and the impact parameter b.

Homework 5. Due Tuesday, February 20, 12:45pm

Problem 1.
Determine the total cross-section for a particle of energy E to "fall" to the center of a field (capture cross-section) $U = -\frac{\alpha}{r^2}$, where $\alpha > 0$.

Problem 2.
Determine the total cross-section for a particle of energy E to "fall" to the center of a field (capture cross-section) $U = -\frac{\alpha}{r^n}$, where $\alpha > 0$ and $n > 2$.

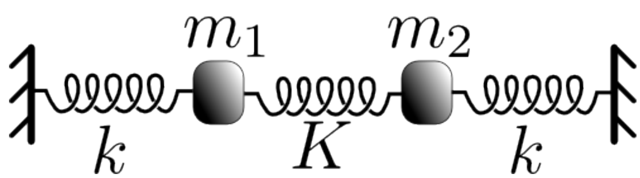
Homework 6. Due Tuesday, February 27, 12:45pm

Problem 1. *1983-Fall-CM-U-2.*
Take $K = 4k$ and $m_1 = m_2 = M$. At $t = 0$ both masses are at their equilibrium positions, m_1 has a velocity v_0 to the right, and m_2 is at rest. Determine

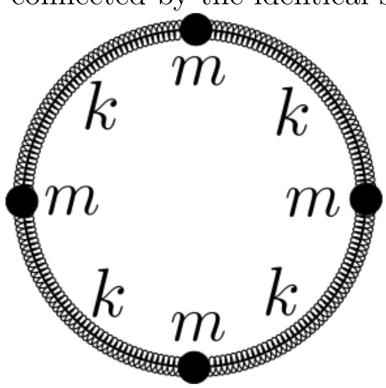
1. The distance, x_1, of m_1 from its equilibrium position at time $t = \frac{\pi}{4}\sqrt{\frac{M}{k}}$.

2. The ratio $\frac{x_1(t_0)}{x_2(t_0)}$, where $t_0 = \frac{\pi}{4}\sqrt{\frac{M}{k}}$.

Hint: First find the normal modes and the normal mode frequencies, then put in the initial conditions.



Problem 2.
Determine the normal/eigen frequencies for the four equal masses m on a ring. The masses are connected by the identical springs of spring constant k along the ring.



For Honors.

Problem 3.
Determine the normal/eigen frequencies for two equal masses m on a ring of radius R. The masses are connected by the identical springs of spring constant k along the ring. The ring is in the **vertical plane**. The acceleration of the free fall is g. Consider only the case of hard springs: $\frac{mg}{kR} \ll 1$.

Homework 7. Due Tuesday, March 05, 12:45pm

Problem 1. *1985-Fall-CM-U-2.*
A straight rod of length b and weight W is composed of two pieces of equal length and cross section joined end-to-end. The densities of the two pieces are 9 and 1. The rod is placed in a smooth, fixed hemispherical bowl of radius R. ($b < 2R$).

1. Find expression for the fixed angle β between the rod and the radius shown in Fig.1
2. Find the position of the center of mass when the rod is horizontal with its denser side on the left (Fig. 1). Give your answer as a distance from the left end.
3. Show that the angle θ which the rod makes with the horizontal when it is in equilibrium (Fig. 2) satisfies

   $$\tan\theta = \frac{2}{5}\frac{1}{\sqrt{(2R/b)^2 - 1}}$$

   Note the fundamental principles you employ in this proof.
4. Show that the equilibrium is stable under small displacements.

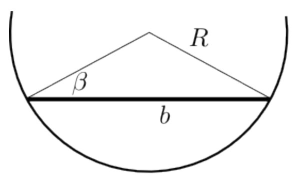


Fig. 1

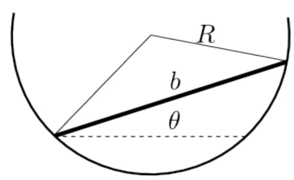
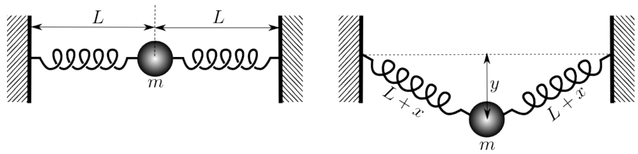


Fig. 2

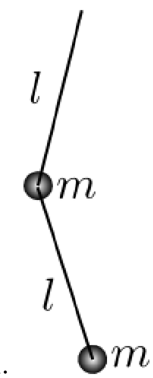
Problem 2. *1990-Fall-CM-U-1.*
A small steel ball with mass m is originally held in place by hand and is connected to two identical horizontal springs fixed to walls as shown in the left figure. The two springs are **unstretched** with natural length L and spring constant k, If the ball is now let go, it will begin to drop and when it is at a distance y below its original position each spring will stretch by an amount x as shown in the right figure. It is observed that the amount of stretching x is very small in comparison to L.

1. Write down the equation that determines $y(t)$. (Take y to be positive in going downward.) Is this simple harmonic motion?
2. Find the equilibrium position y_{eq} about which the steel ball will oscillate in terms of m, g, k, and L.
3. Find the maximum distance, y_{max}, that the steel ball can drop below its original position in therms of m, g, k, and L.
4. Write down an expression for the period of the steel ball's motion. (DO NOT evaluate the integral.)


L
L
m
y
L + x
L + x
m

Homework 8. Due Tuesday, March 19, 12:45pm

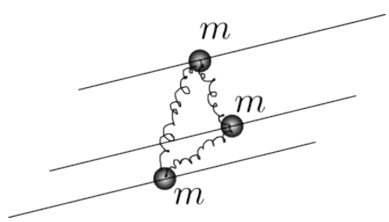
Problem 1.



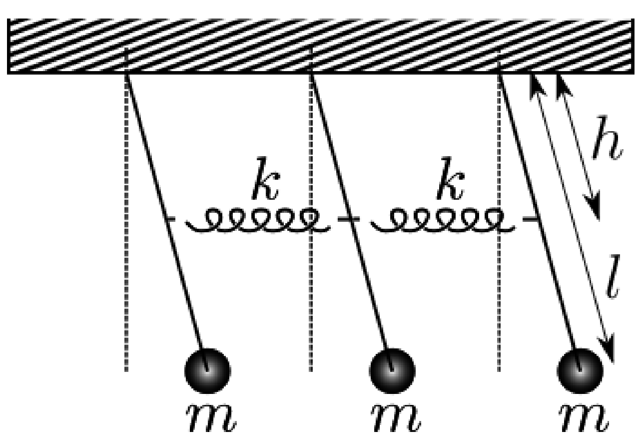
Find normal frequencies and normal modes of a double pendulum.

Problem 2.
Three infinite horizontal rails are at distance l from each other. Three beads of equal masses m can slide along the rails without friction. The three beads are connected by identical springs of constants k and equilibrium/unstretched length $l/2$.

1. Find the normal modes and the normal frequencies.
2. At time $t = 0$ one of the masses is suddenly given a velocity v_0. Find how the velocities of all three masses will depend on time. Find how the sum of the three velocities depends on time. Why?

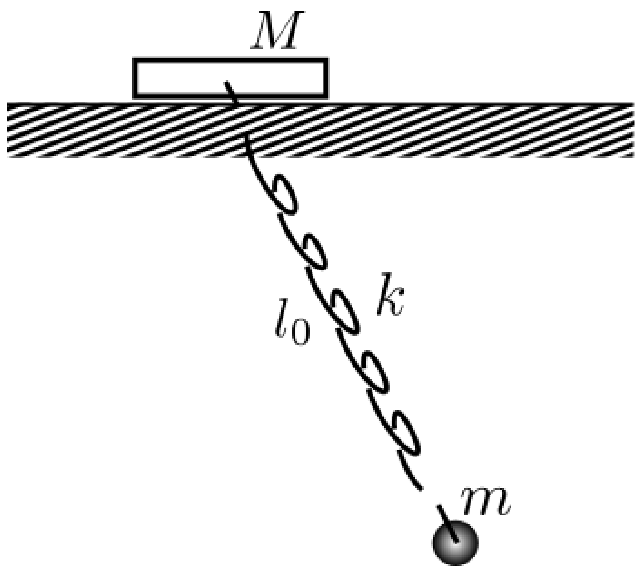


Problem 3. *Three pendulums*
Three identical pendulums are connected by the springs as shown on the figure. Find the normal modes



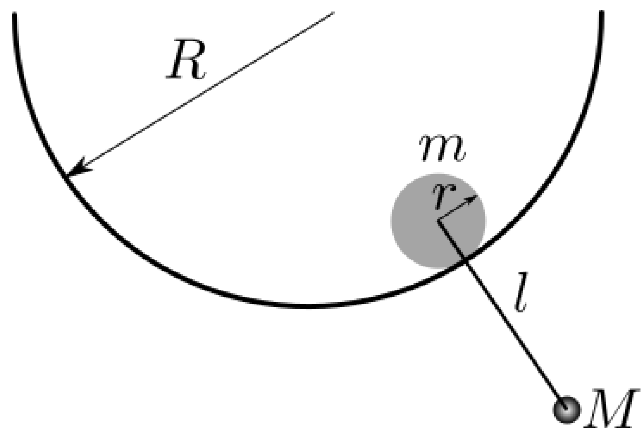
Problem 4. *A pendulum a spring and a block*

A ball of mass m is hanging on a weightless spring of spring constant k, which is attached to a block of mass M. The block can move along the table without friction. The length of the unstretched spring is l_0. Find the normal modes of small oscillations.

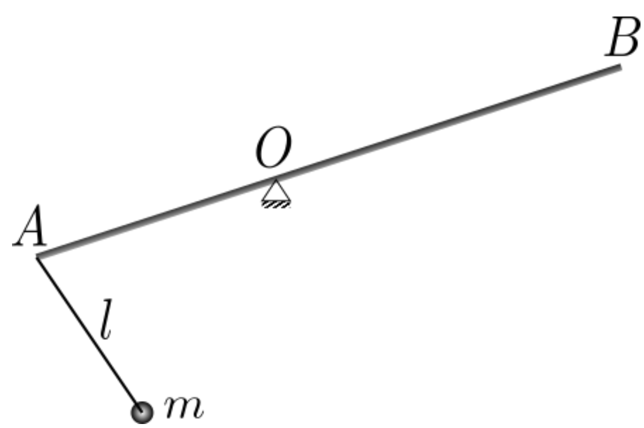


Homework 9. Due Tuesday, March 26, 12:45pm

Problem 1. *Cylinders and a pendulum*
A uniform solid cylinder of radius r and mass m can roll inside a hollow cylinder of radius $R > r$ without slipping. A pendulum of length $l = \frac{R-r}{2}$ and mass $M = m/2$ is attached to the center of the smaller cylinder. Find the normal frequencies and normal modes of this system.



Problem 2. *A rod and a pendulum*
A uniform rod AB of mass M and length $3a$ can rotate around a point O, $|AO| = a$. A pendulum of length $l = a$ and mass $m = \frac{3}{4}M$ is attached to the end A of the rod. Find the normal frequencies and normal modes.



Problem 3. *One ring to rule them all.*
N identical beads (mass m) can move along the ring of radius R. Each bead is connected to the one behind it and to the one in front of it by a spring of constant k. Find all normal frequencies.

Homework 10. Due Tuesday, April 2, 12:45pm

Problem 1.

The center is in the center of mass.

1. Find the principal moments of inertia for a uniform square of mass M and side a.

2. Find the kinetic energy of the uniform cube of mass M and side a rotating with angular velocity $\vec{\Omega}$ around an arbitrary axis which goes through the center of mass of the cube.

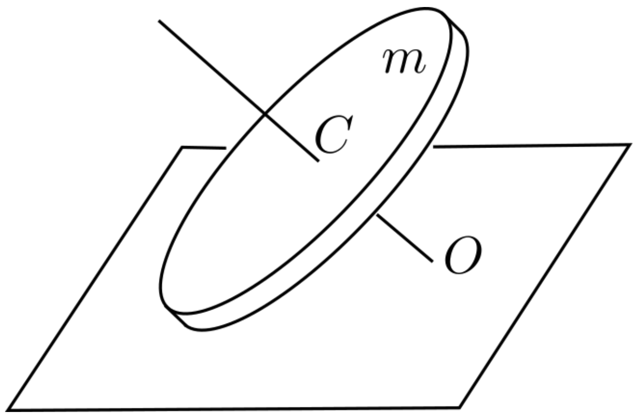
Problem 2.

1. Find the kinetic energy of the uniform circular cone of height h, base radius R, and mass M. Rotating with the angular velocity $\vec{\Omega}$ around an arbitrary axis which goes through the center of mass.

2. The same for a uniform ellipsoid of semiaxes a, b, and c.

Homework 11. Due Tuesday, April 9, 12:45pm

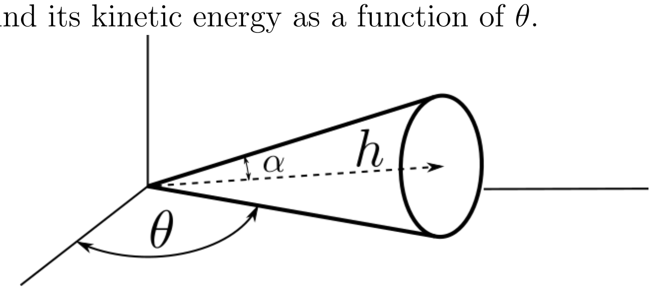
Problem 1. *A coin*
A uniform thin disc of mass m rolls without slipping on a horizontal plane. The disc makes an angle α with the plane, the center of the disc C moves around the horizontal circle with a constant speed v. The axis of symmetry of the disc CO intersects the horizontal plane in the point O. The point O is stationary. Find the kinetic energy of the disc.



Problem 2.
If we ignore the interaction between the Earth and both the Moon and the Sun, the celestial pole, as observed from the Earth, is traveling on a small circle in the sky. Find the period (in days) of this travel of the celestial pole. Consider Earth to be a uniform spheroid with semiaxes C and A and $\frac{C-A}{A} \approx \frac{1}{300}$.

For Honors.

Problem 3.
A cylindrical cone of mass M, height h, and angle α is rolling on the plane without slipping. Find its kinetic energy as a function of $\dot{\theta}$.



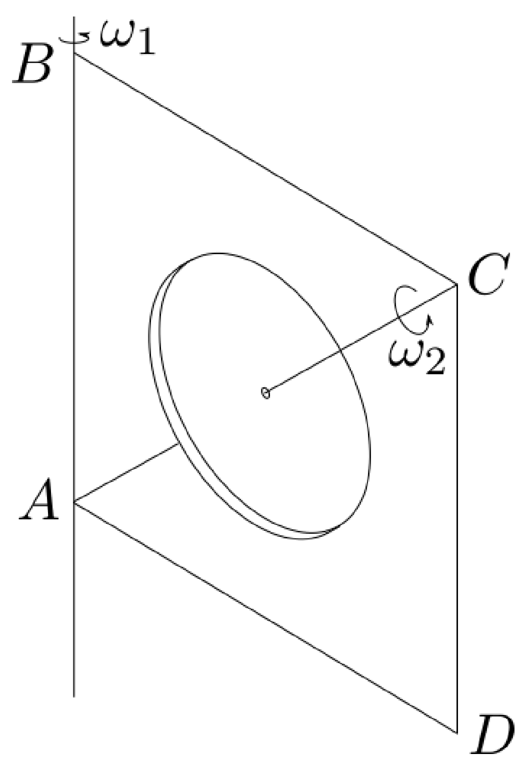
Homework 12. Due Tuesday, April 16, 12:45pm

Problem 1. *A Rectangular Parallelepiped*
A uniform Rectangular Parallelepiped of mass m and edges a, b, and c is rotating with the constant angular velocity ω around an axis which coincides with the parallelepiped's large diagonal.

1. What is the parallelepiped's kinetic energy?
2. What torque must be applied to the axis of rotation in order to keep the axis(!) still? (neglect the gravity.)

Problem 2. *A disk in a frame*
A square weightless frame $ABCD$ with the side a is rotating around the side AB with the constant angular velocity ω_1. A uniform disk of radius R and mass m is rotating around the frame's diagonal AC with the constant angular velocity ω_2. The center of the disk coincides with the center of the frame. Assuming $\omega_1 = \omega_2 = \omega$ find

1. The kinetic energy of the system.
2. The magnitude of the torque that needs to be applied to the axis AB to keep it (the axis AB) still. (neglect the gravity.)

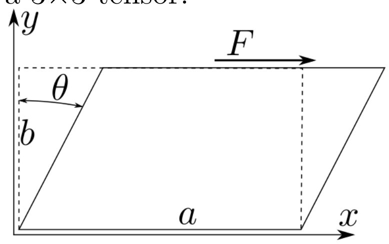


Hint: Angular velocity is a vector!

Homework 13. THE LAST ONE! Due Tuesday, April 23, 12:45pm

Problem 1.

A parallelepiped with edges a, b, c was deformed as shown on the figure. Find the strain tensor in the linear approximation assuming that θ is **small**. Remember, the answer must be a 3×3 tensor.



Problem 2.

For the deformation from the previous problem assuming that the sheer modulus is μ find:

1. The stress tensor.
2. The force F which has to be applied to the top surface.
3. Total torque of **ALL** external forces.
4. Are you sure your answer for the previous question is correct?
5. How come the parallelepiped is not rotating then?