EXAM 1. Tuesday, March 22-24, 2022, take home. Due on Thursday, March 24, 12:45pm.

Problem 1. *1984-Spring-CM-U-2.*
A particle of mass m moves subject to a central force whose potential is $V(r) = Kr^3$, $K > 0$.

1. For what kinetic energy and angular momentum will the orbit be a circle of radius a about the origin?

2. What is the period T of this circular motion?

3. If the motion is slightly disturbed from this circular orbit, what will be the period τ of small radial oscillations about $r = a$? Express τ through T. (Assume that the disturbance is in the radial direction.)

Problem 2. *1990-Fall-CM-U-3*
A large sphere of radius R and mass M has a mass density that varies according to the distance from the center, r:

$$\rho(r) = \begin{cases} \rho_0 \left[1 - (r/R)^n\right], & \text{if } r \leq R; \\ 0, & \text{if } r > R. \end{cases},$$

where $n > 0$. A very small hole is drilled through the center of the sphere and a small object of mass m is released from rest into the hole at the surface. How fast will the object be moving when it reaches the center of the sphere? Express your answer through M, R, n, and G.

Problem 3. *1991-Fall-CM-U-2.*
A steel ball of mass M is attached by massless springs to four corners of a $2a$ by $b+c$ horizontal, rectangular frame. The springs constants k_1 and k_2 with corresponding equilibrium lengths $L_1 = \sqrt{a^2 + b^2}$ and $L_2 = \sqrt{a^2 + c^2}$ as shown. Neglecting the force of gravity,

1. Find the frequencies of small amplitude oscillations of the steel ball in the plane of rectangular frame.

2. Identify the type of motion associated with each frequency.

3. Is the oscillation of the steel ball perpendicular to the plane of the rectangular frame harmonic? Why or why not?
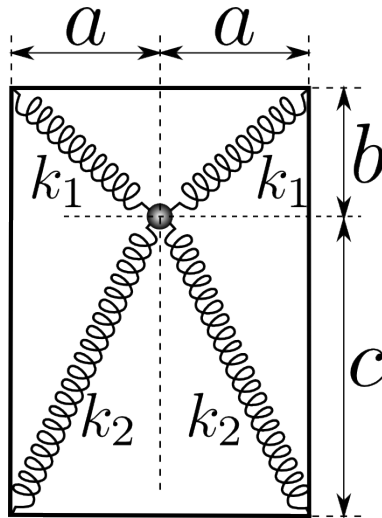
Problem 4. *Paraboloid scattering.*

An immovable paraboloid (which was produced by rotating a parabola $y = \frac{x^2}{4R}$ around the y axis), elastically scatters particles that are coming from $y = -\infty$. Find the differential scattering cross-section $d\sigma$.