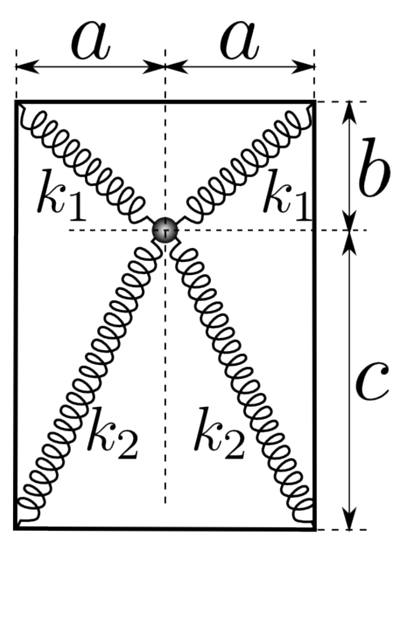
EXAM 1. Tuesday, March 23-25, 2021, take home. Due on Thursday, March 25, 1:30pm.

Problem 1. *1984-Spring-CM-U-2.*
A particle of mass m moves subject to a central force whose potential is $V(r) = Kr^3$, $K > 0$.

1. For what kinetic energy and angular momentum will the orbit be a circle of radius a about the origin?
2. What is the period T of this circular motion?
3. If the motion is slightly disturbed from this circular orbit, what will be the period τ of small radial oscillations about $r = a$? Express τ through T. (Assume that the disturbance is in the radial direction.)

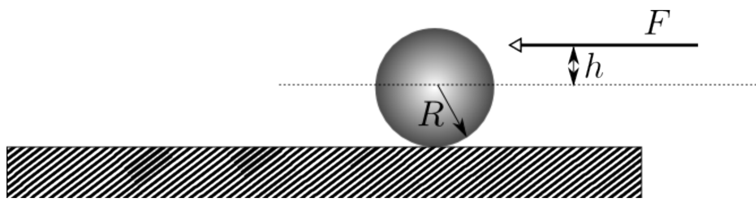
Problem 2. *1991-Fall-CM-U-2.*
A steel ball of mass M is attached by massless springs to four corners of a $2a$ by $b+c$ horizontal, rectangular frame. The springs constants k_1 and k_2 with corresponding equilibrium lengths $L_1 = \sqrt{a^2 + b^2}$ and $L_2 = \sqrt{a^2 + c^2}$ as shown. Neglecting the force of gravity,

1. Find the frequencies of small amplitude oscillations of the steel ball in the plane of rectangular frame.
2. Identify the type of motion associated with each frequency.
3. Is the oscillation of the steel ball perpendicular to the plane of the rectangular frame harmonic? Why or why not?



Problem 3. *1996-Spring-CM-U-1*

A billiard ball initially at rest, is given a sharp impulse by a cue stick. The cue stick is held horizontally a distance h above the centerline as in the figure below. The ball leaves the cue with a horizontal speed v_0 and, due to its spin, eventually reaches a final speed of $9v_0/7$. Find h in terms of R, where R is the radius of the ball. You may assume that the impulsive force F is much larger than the frictional force during the short time that the impulse is acting.



Problem 4. *2001-Spring-CM-U-2*

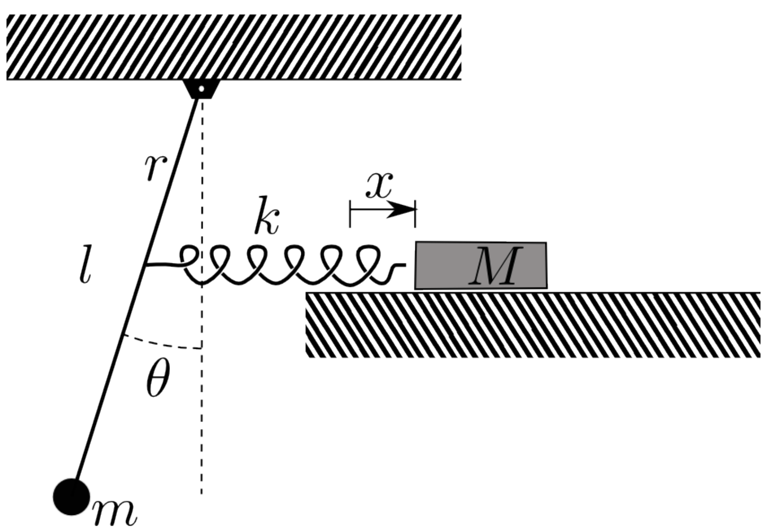
Consider the system, pictured below, which consists of a ball of mass m connected to a massless rod of length l. This is then joined at point r to a spring of spring constant k connected to a block of mass M which rests on a frictionless table. When $\theta = 0$ and $x = 0$ the spring is unstretched.

1. Write the Lagrangian for the system in terms of the coordinates θ and x assuming small displacements of the pendulum

2. Write the equations of motion for the system.

3. Making the simplifying assumptions that $M = m$, $l = 2r$, and setting $k/m = g/l = \omega_0^2$, find the normal-mode frequencies of this system for small oscillations in terms of ω_0

4. Assuming the same conditions, calculate the ratio of amplitudes (for each of the two masses) of the two normal modes of oscillation. In other words, find the normal modes.



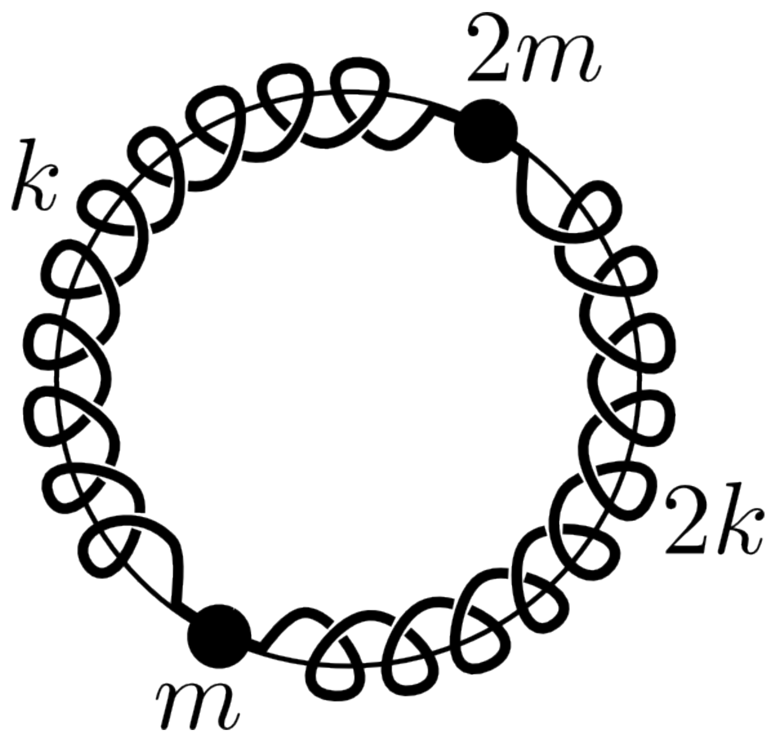
EXAM 2. Final. Thursday, May 6, 2021, 8:00-10:30am

Problem 1. *1998-Spring-CM-U-1*
The system shown below consists of two small beads, one of mass m and the other of mass $2m$, that are both free to slide on the horizontal frictionless fixed ring of radius R. The beads are connected by springs, one with spring constant k and the other with spring constant $2k$, that wind around the ring as shown. The equilibrium length of each of the springs is πR.

1. Find equations of motion for the two beads. Be sure to draw a figure that clearly defines your choice of generalized coordinates.

2. Find the frequencies of small oscillation of the system and the normal mode associated with each of the small oscillation frequencies.

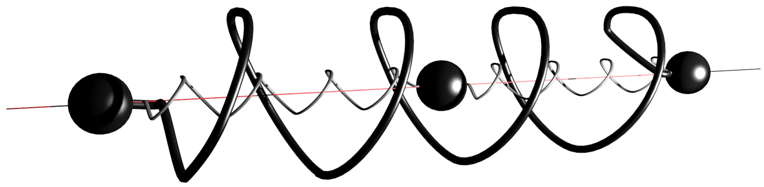
3. Give a simple physical interpretation of the frequencies and normal modes that you found in the previous part.



Problem 2. *2001-Fall-CM-U-3*
Three spheres of equal mass m are constrained to move in one dimension along the line connecting their centers. The three spheres are connected by three springs, as shown in the figure. The three springs have equal spring constants k In equilibrium, all three of the springs are at their respective natural lengths.

1. Choose a reasonable set of coordinates and find the equations of motion.

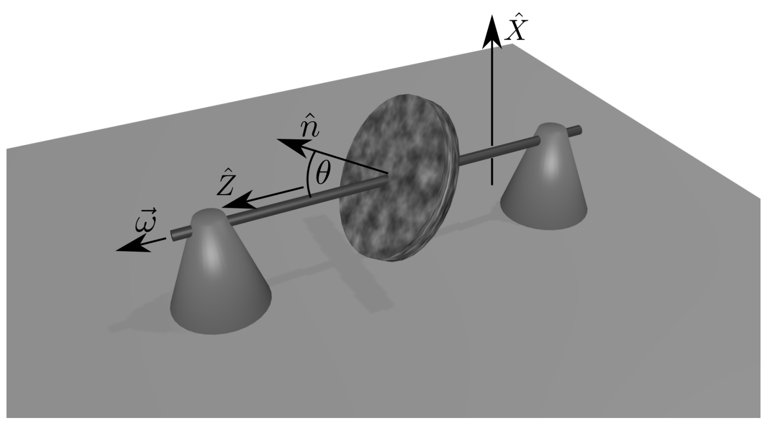
2. Find the normal-mode frequencies.



Problem 3. *Disc at an angle*

A disk is rigidly attached to an massless axle passing through its center so that the disc's symmetry axis $\hat{n}$ makes an angle θ with the axle. The moments of inertia of the disc relative to its center are C about the symmetry axis $\hat{n}$ and A about any direction $\hat{n}'$ perpendicular to $\hat{n}$. The axle spins with constant angular velocity $\vec{\omega} = \omega\hat{Z}$ ($\hat{Z}$ is a unit vector along the axle.)

1. What is the kinetic energy of the disc?
2. What is the angular momentum, $\vec{L}$, expressed in the internal frame of reference of principle axes of inertia.
3. Find the torque, $\vec{\tau}$, which must be exerted on the axle by the bearings which support it. Specify the components of $\vec{\tau}$ along the principle axes of inertia of the disc.



THE END!