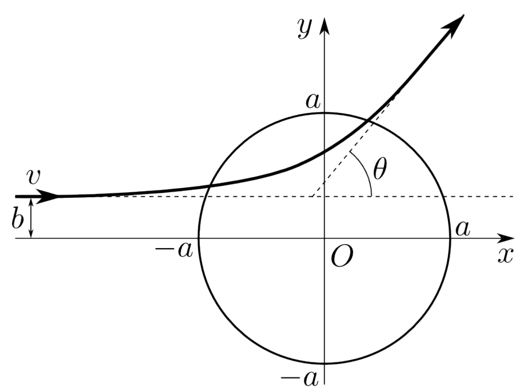
EXAM 1. Tuesday, March 24-26, 2020, take home

Problem 1. *2000-Fall-CM-G-5.jpg*
Particles are scattered classically by a potential:

$$V(r) = \begin{cases} U(1 - r^2/a^2), & \text{for } r \le a \\ 0, & \text{for } r > a \end{cases}, \quad U \text{ is a constant.}$$

Assume that $U > 0$. A particle of mass m is coming in from the left with initial velocity v_0 and impact parameter $b < a$. Hint: work in coordinates (x, y) not (r, ϕ).

1. What are the equations of motion for determining the trajectory $x(t)$ and $y(t)$ when $r < a$?
2. Assume that at $t = 0$ the particle is at the boundary of the potential $r = a$. Solve your equations from the previous part to find the trajectory $x(t)$ and $y(t)$ for the time period when $r < a$. Express your answer in terms of sinh and cosh functions.
3. For initial energy $\frac{1}{2}mv_0^2 = U$, find the scattering angle θ as function of b.



Problem 2. *1984-Spring-CM-U-2.*
A particle of mass m moves subject to a central force whose potential is $V(r) = Kr^3$, $K > 0$.

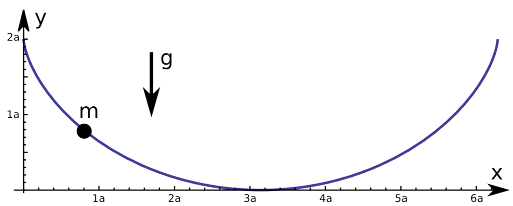
1. For what kinetic energy and angular momentum will the orbit be a circle of radius a about the origin?
2. What is the period T of this circular motion?
3. If the motion is slightly disturbed from this circular orbit, what will be the period τ of small radial oscillations about $r = a$? Express τ through T.

Problem 3. *1999-Spring-CM-U-1*
A bead of mass m moves on a fixed frictionless wire shaped as a cycloid:

$$x = a(\theta - \sin\theta)$$
$$y = a(1 + \cos\theta)$$

The wire is oriented in a vertical plane, with the $+y$ direction pointing upward and the gravitational force downward.

1. Find the differential equation(s) of motion for the bead, but do not solve the equation(s).
2. Find the frequency of small amplitude oscillations of the bead on the wire about the equilibrium location.



Problem 4. *2001-Spring-CM-U-2*
Consider the system, pictured below, which consists of a ball of mass m connected to a massless rod of length l. This is then joined at point r to a spring of spring constant k connected to a block of mass M which rests on a frictionless table. When $\theta = 0$ and $x = 0$ the spring is unstretched.

1. Write the Lagrangian for the system in terms of the coordinates θ and x assuming small displacements of the pendulum
2. Write the equations of motion for the system.
3. Making the simplifying assumptions that $M = m$, $l = 2r$, and setting $k/m = g/l = \omega_0^2$, find the normal-mode frequencies of this system for small oscillations in terms of ω_0
4. Assuming the same conditions, calculate the ratio of amplitudes (for each of the two masses) of the two normal modes of oscillation.