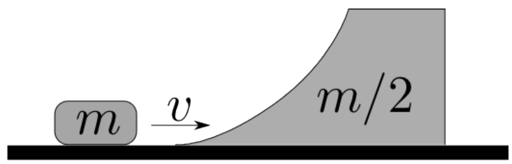
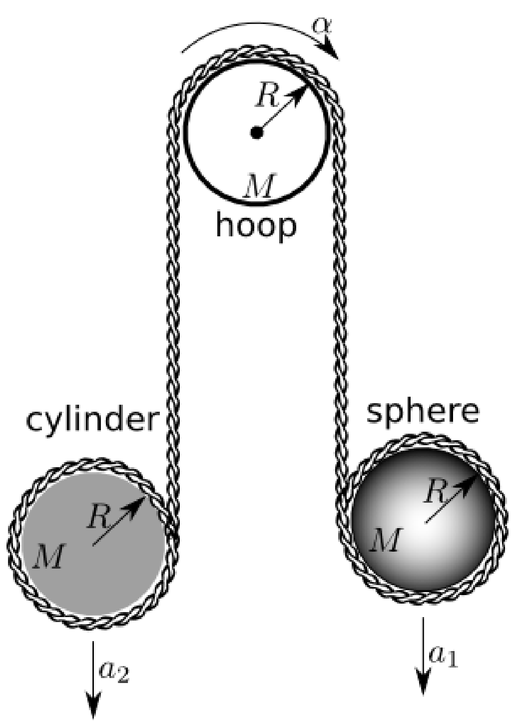
EXAM 1.

Problem 1. *1987-Fall-CM-U-1.*
A block of mass m slides on a frictionless table with velocity v. At $x = 0$, it encounters a frictionless ramp of mass $m/2$ which is sitting at rest on the frictionless table. The block slides up the ramp, reaches maximum height, and slides back down.

1. What is the velocity of the block when it reaches its maximum height?
2. How high above the frictionless table does the block rise?
3. What are the final velocities of the block and the ramp?



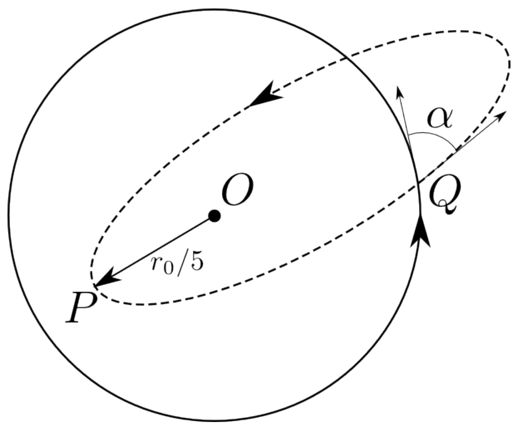
Problem 2. *1987-Fall-CM-U-2.jpg*
A massless rope winds around a cylinder of mass M and radius R, over a pulley, and then wraps around a solid sphere of mass M and radius R. The pulley is a hoop, also of mass M and radius R, and is free to turn about a frictionless bearing located at its center. The rope does not slip on the pulley. Find the linear accelerations a_1 and a_2 of the centers of the sphere and cylinder respectively, and the angular acceleration α of the pulley. The positive directions for a_1, a_2, and α are shown in the figure.



Problem 3. *1993-Fall-CM-U-1*

A satellite of mass m is traveling at speed v_0 in a circular orbit of radius r_0 under the gravitational force of a fixed mass M at point O. At a certain point Q in the orbit (see the figure below) the direction of motion of the satellite is suddenly changed by an angle α without any change in the magnitude of the velocity. As a result the satellite goes into an elliptic orbit. Its distance of the closest approach to O (at point P) is $r_0/5$.

1. What is the speed of the satellite at P, expressed as a multiple of v_0?
2. Find the angle α.



Problem 4. *1993-Fall-CM-U-3.*

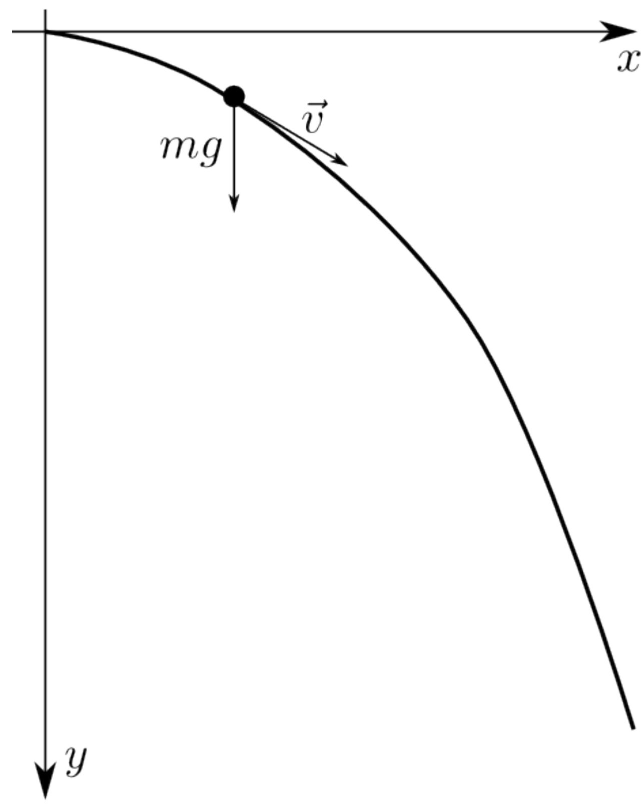
A point-like test mass m is placed at the center of a spherical planet of uniform density, mass M and radius R.

1. Calculate the energy required to remove this test mass to an infinite distance from the planet.
2. Calculate the gravitational binding energy of this planet (without the test mass), i.e. the energy required to break the planet up into infinitesimal pieces separated by an infinite distance from each other.

Problem 5. *1996-Fall-CM-U-2*

A particle of mass m slides down a curve $y = kx^2$, $(k > 0)$ under the influence of gravity, as illustrated. There is no friction, and the particle is constrained to stay on the curve. It starts from the top with negligible velocity.

1. Find the velocity v as a function of x.
2. Next, assume that the particle initially slides down the curve under gravity, but this time is not constrained to the curve. Does it leave the curve after it has fallen a certain distance? Prove your answer.

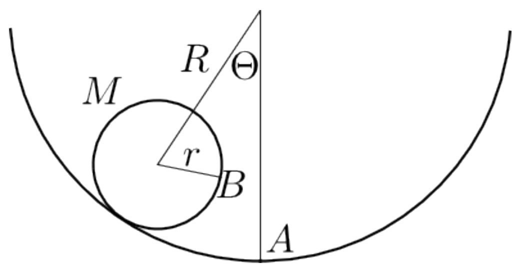


x
$\vec{v}$
mg
y

EXAM 2. Final. Monday, December 10, 2018, 8-10am

Problem 1. *1983-Fall-CM-U-3.*
A hollow thin walled cylinder of radius r and mass M is constrained to roll without slipping inside a cylindrical surface with radius $R+r$ (see diagram). The point B coincides with the point A when the cylinder has its minimum potential energy.

1. What is the frequency of small oscillations around the equilibrium position?
2. What would the frequency of small oscillations be if the contact between the surfaces is frictionless?



Problem 2. *1984-Spring-CM-U-2.*
A particle of mass m moves subject to a central force whose potential is $V(r) = Kr^3$, $K > 0$.

1. For what kinetic energy and angular momentum will the orbit be a circle of radius a about the origin?
2. What is the period T of this circular motion?
3. If the motion is slightly disturbed from this circular orbit, what will be the period τ of small radial oscillations about $r = a$? Express τ through T.

Problem 3. *1991-Spring-CM-U-2.*
A circular platform of mass M and radius R is free to rotate about a vertical axis through its center. A man of mass M is originally standing right at the edge of the platform at the end of a line painted along a diameter of the platform. The platform and man are set spinning with an angular velocity ω_0. At $t = 0$ the man begins to walk toward the center of the platform along the line so that his distance from the center is $R - v_0 t$. If the man slips off the line when he is at $R/2$, what must be the coefficient of friction between the man and the platform?