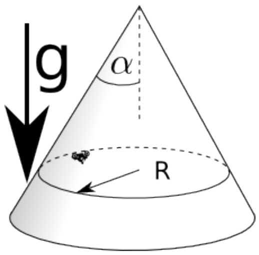
EXAM 1. Friday, October 3, 2014, 7-9pm

Problem 1. *Crazy biker.*



A biker is riding on the inner surface of the cone of angle α along the circle of the radius R with angular velocity ω. What should the coefficient of friction μ be in order for the biker to not fall down?

Problem 2. *1985-Spring-CM-U-2.*

Two cylinders having radii R_1 and R_2 and rotational inertias I_1 and I_2 respectively, are supported by fixed axes perpendicular to the plane of the figure. The large cylinder is initially rotating with angular velocity ω_0. The small cylinder is moved to the right until it touches the large cylinder and is caused to rotate by the frictional force between the two. Eventually, slipping ceases, and the two cylinders rotate at constant rates in opposite directions. Find the final angular velocity ω_2 of the small cylinder in terms of I_1, I_2, R_1, R_2, and ω_0.

Problem 3. *1990-Fall-CM-U-3.jpg*

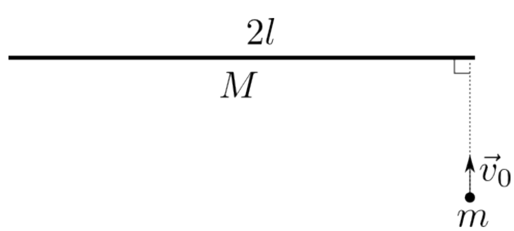
A large sphere of radius R and mass M has a mass density that varies according to the distance from the center, r:

$$\rho(r) = \begin{cases} \rho_0 \left(1 - r/R\right), & \text{if } r \leq R; \\ 0, & \text{if } r > R. \end{cases}$$

A very small hole is drilled through the center of the sphere and a small object of mass m is released from rest into the hole at the surface. How fast will the object be moving when it reaches the center of the sphere?

Problem 4. *1995-Spring-CM-U-3*

A uniform line-like bar of mass M, and length $2l$ rests on a frictionless, horizontal table. A point-like particle of mass m slides along the table with velocity v_0 perpendicular to the bar and strikes the bar very near one end, as illustrated below. Assume that the force between the bar and the particle during the collision is in the plane of the table and perpendicular to the bar. If the interaction is elastic (*i.e.*, if energy is conserved) and lasts an infinitesimal amount of time, then determine the rod's center-of-mass velocity V and angular velocity ω, and the particle's velocity v after the collision.



Problem 5. **Extra for Honors**

A soap bubble has a radius R. The surface tension of the soap film is σ. What is the air pressure difference inside and outside the bubble?

EXAM 2. Friday, November 7, 2014, 7-9pm

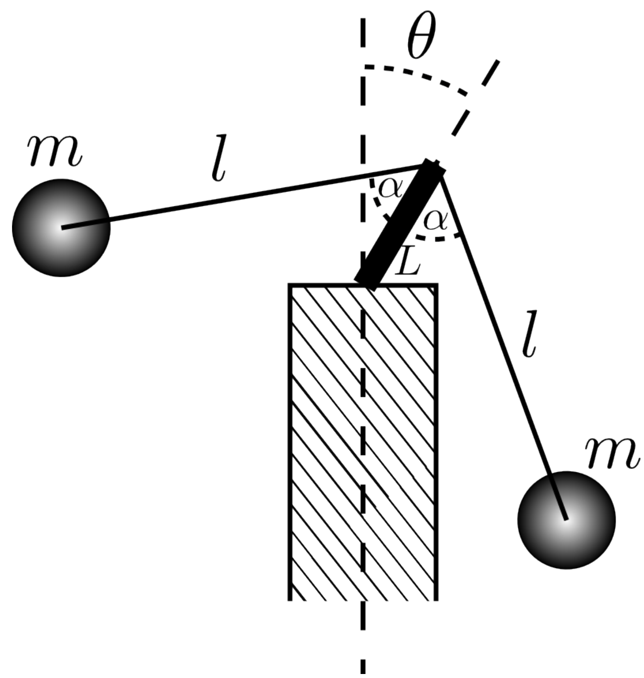
Problem 1. *1 point*
Write down your name. Clearly. In big readable letters.

Problem 2. *1992-Fall-CM-U-2.*
A toy consists of two equal masses (m) which hang from straight massless arms (length l) from an essentially massless pin. The pin (length L) and the arms are in plane. Consider only motion in this plane.

1. Find the potential and kinetic energies of the masses as a function of θ, the angle between the vertical and the pin, and the time derivatives of θ. (Assume the toy is rocking back and forth about the pivot point.)

2. Find the condition in terms of L, l, and α such that this device is stable.

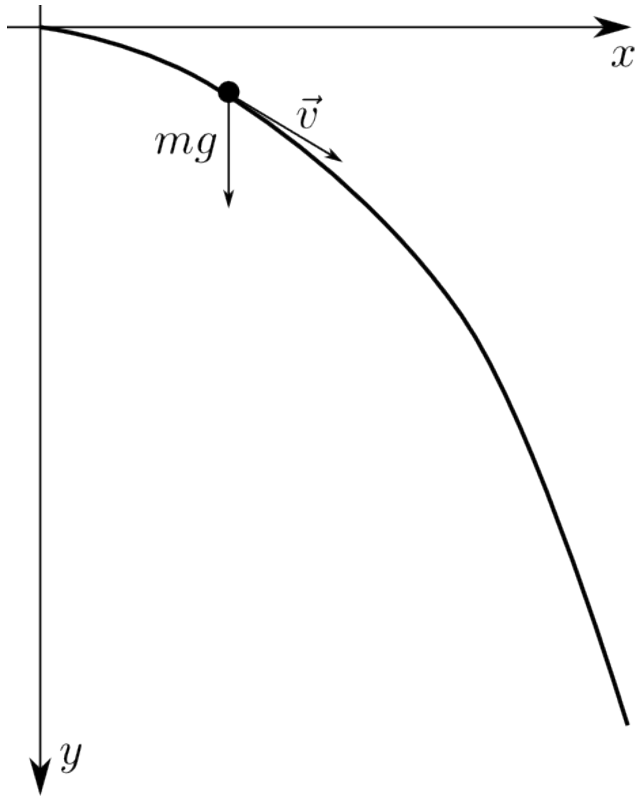
3. Find the period of oscillation if θ is restricted to very small values.



Problem 3. *1996-Fall-CM-U-2*
A particle of mass m slides down a curve $y = kx^2$, ($k > 0$) under the influence of gravity, as illustrated. There is no friction, and the particle is constrained to stay on the curve. It starts from the top with negligible velocity.

1. Find the velocity v as a function of x.

2. Next, assume that the particle initially slides down the curve under gravity, but this time is not constrained to the curve. Does it leave the curve after it has fallen a certain distance? Prove your answer.



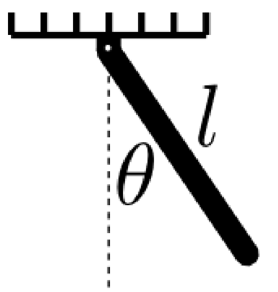
Problem 4. *1999-Fall-CM-U-2.*
A particle of mass m moves in a circular orbit of radius r_0 under the influence of a potential $V(r) = V_0 \ln(r/r_0)$, where $V_0 > 0$.

1. Find the period of the orbit.
2. Find the frequency of small (radial) oscillations about the circular orbit.
3. Show that, while the circular orbit is a stable solution, the orbit with small radial oscillations is not periodic.

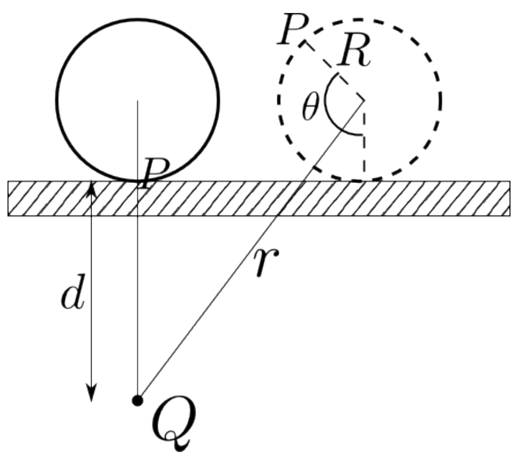
EXAM 3. Final. Tuesday, December 16, 2014, 8-10am

Problem 1. *1986-Spring-CM-U-2.*
A "physical pendulum" is constructed by hanging a thin uniform rod of length l and mass m from the ceiling as shown in the figure. The hinge at ceiling is frictionless and constrains the rod to swing in a plane. The angle θ is measured from the vertical.

1. Find the Lagrangian for the system.
2. Use Euler-Lagrange differential equation(s) to find the equation(s) of motion for the system. (BUT DON'T SOLVE).
3. Find the approximate solution of the Euler-Lagrange differential equation(s) for the case in which the maximum value of θ is small.
4. Find the Hamiltonian $H(p, q)$ for the system.
5. Use the canonical equations of Hamilton to find the equations of motion for the system and solve for the case of small maximum angle θ. Compare your results with b. and c.



Problem 2. *1992-Fall-CM-U-3.*
A homogeneous disk of radius R and mass M rolls without slipping on a horizontal surface and is attracted to a point Q which lies a distance d below the plane (see figure). If the force of attraction is proportional to the distance from the center of mass of the disk to the force center Q, find the frequency of oscillations about the position of equilibrium using the Lagrangian formulation.



Problem 3. *1997-Spring-CM-U-2*
A particle of mass m moves in a region where its potential energy is given by

$$V = Cr^4,$$

where C is a real, positive constant. Consider the case where the particle moves in a circular orbit of radius R.

1. Express its total energy E and angular momentum L as a function of R.
2. Determine the period τ_{orb}, of this circular orbit as a function of R.
3. What is its period τ_{rad} for small radial oscillations if the orbit is slightly perturbed? Express τ_{rad} as a factor times τ_{orb}.

Problem 4. *1999-Fall-CM-U-3.*
A mass m slides down the outside of horizontal cylinder of radius R. Its initial velocity at the top of the cylinder ($\theta = 0$) is very small and can be neglected, and the whole motion takes place in a vertical plane. There is no friction between the mass and the cylinder, but there is a force opposing to the motion due to air resistance. The force is antiparallel to the velocity v and has magnitude $\beta m v^2/2R$, where β is a dimensionless parameter.

1. Show that the angle at which the mass leaves the cylinder is given by:

   $$(1+\beta^2)\cos\theta = 2\left(e^{-\beta\theta} - \cos\theta + \beta\sin\theta\right)$$

   [**Hint:** After substitution into your equation of motion, and subsequent multiplication by an appropriate function, you can obtain an integrable differential equation.]

2. Use the result from part **a.** to derive an approximate value needed for β such that the angle at which the mass leaves the cylinder is $1/100$ of a radian less then $\pi/2$.