

Homework 1. Due Friday, June 2, 10:00 am.

Problem 1. *Cannon*

A cannon can shoot whatever cannons shoot at different angles to the horizon, but with the same initial velocity. At what angle does the cannon shoot to a maximal distance? What would be that angle on the moon?

Problem 2. *Colliding particles*

A point like particle collides with the identical particle which is initially at rest. The particles repel each other with the central force given by $\vec{F} = G(\vec{r}_1 - \vec{r}_2)e^{-(\vec{r}_1 - \vec{r}_2)^2/2l^2}$, where l and G are some constants and \vec{r}_1 and \vec{r}_2 are the positions of the particles. Find the angle between the particles' velocities after a long time after collision. Neglect the gravity.

Problem 3. *Rising Snake*

A snake of length L and linear mass density ρ rises from the table from rest. It pushes its head straight up with the constant velocity v . The part of the snake which is on the table is not moving.

1. What is the normal force that the snake exert on the table?
2. What is the tangential (parallel to the surface of the table) force that the snake exert on the table?
3. What is the minimal coefficient of friction between the snake and the table which would allow the snake to raise like that?
4. What is the total mechanical work which the snake will have done when it rises full length?

Next problem is an extra problem. It will give you 5 points to the FINAL grade. Due by Friday June 23, 10:00 am. Please, do not upload the solution to the Canvas. Instead send it to me by email.

Problem 4. *Boat in a lake.*

A boat floats in a lake. When the boat moves the force of resistance is proportional to the velocity of the boat. Initially the boat is at rest. A person walks from the stern to the bow of the boat. What will be the position of the boat (relative to the initial position of the boat) long time after the person stopped moving?

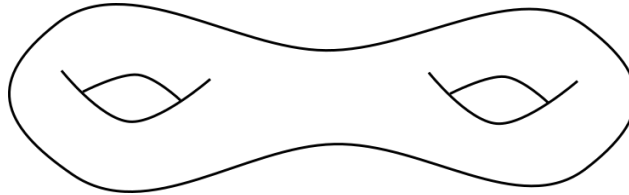
Homework 2. Due Friday, June 9, 10am

Problem 1. *Sum of angles*

On a sphere a sum of all angles of a triangle is larger than π . What is the largest possible sum of all angles of a triangle on a sphere? What is it for a triangle on a hemisphere?

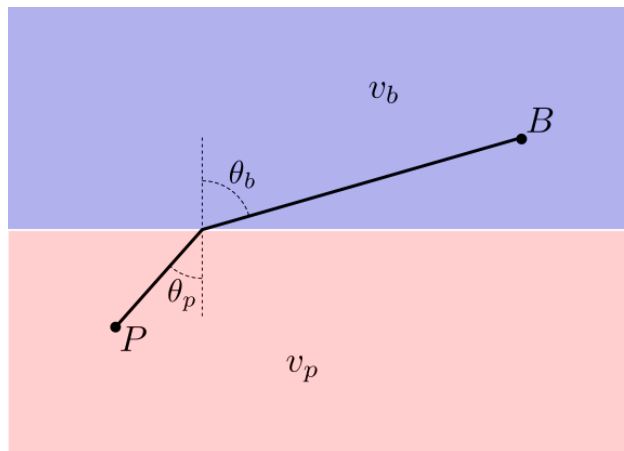
Problem 2. *Double torus*

Approximate a body shown on the figure by a polyhedral and compute $V + F - E$. Does it equal to $2 - 2g$? (Draw the polyhedral and write down V , F , and E .)



Problem 3. *Blue and pink*

A person can run with velocity v_b in the blue region and with velocity v_p in the pink region in the figure. What is the relation between θ_p and θ_b for the path that takes the person the minimal time to run from the point P to the point B ?



Homework 3. Due Friday, June 16, 10am

Problem 1. *Conservative forces*

- Which of the two 2D forces $\vec{F}^A(x, y)$ or $\vec{F}^B(x, y)$ given below is conservative? The x and y components of the force $\vec{F}^A(x, y)$ are given by

$$\begin{aligned}F_x^A(x, y) &= -6x - y \cos(xy) - 14xy, \\F_y^A(x, y) &= -5 - x \cos(xy) - 7x^2\end{aligned}$$

and the x and y components of the force $\vec{F}^B(x, y)$ are given by

$$\begin{aligned}F_x^B(x, y) &= -6x - y \cos(xy) - 12xy, \\F_y^B(x, y) &= -5 - x \cos(xy) - 7x^2.\end{aligned}$$

- Write the potential $U(x, y)$ for the conservative one?

Problem 2. *Free fall*

Derive the formula $x(t) = x_0 + v_0t + \frac{at^2}{2}$ for the 1D motion with constant acceleration a starting from the energy conservation law with potential energy $U(x) = -xma$, deriving the integral representation of the solution, and computing the last integral.

Problem 3. *May the fourth be with you*

A particle of mass m is moving in 1D in a potential $U(x) = kx^4$, where $k > 0$. The total energy of the particle $E > 0$.

- Find the period T of motion of the particle. (You do not need to take the last integral. Just write it.)
- How the period T depends on the total energy E ?
- How the period of motion T depends on the total energy E if the potential energy is given by $U(x) = kx^2$?
- What will happen in both cases if $k < 0$?

Homework 4. Due Friday, June 23, 10am

Problem 1. *A hole in the Earth*

1. A straight hole is drilled from the north pole to the south pole of the Earth. What time will it take for a ball dropped into the hole on the north pole to appear in the south pole? Neglect air resistance. Take the Earth to be a uniform sphere. Express your answer through the acceleration of free fall g and Earth radius R .
2. Compare the time you obtained in the part 1 to the time it take a satellite to go from the north pole to the south pole. The satellite is on circular orbit close to the Earth surface.

Problem 2. *Pendulum*

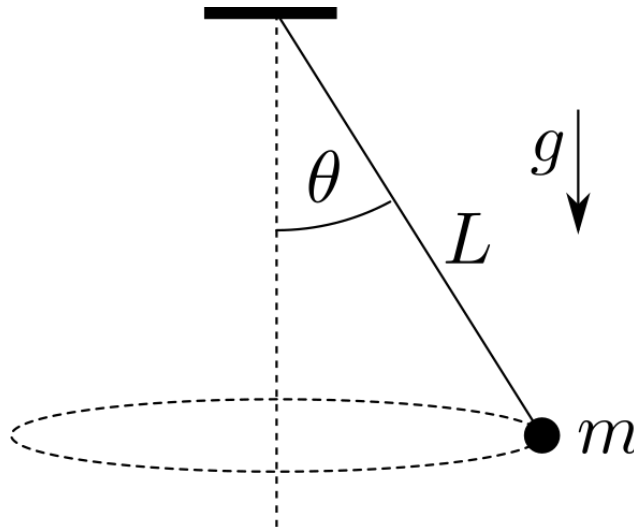
A simple pendulum of length l and mass m is oscillating in the Earth gravitational field. Using its angle ϕ as a coordinate.

1. Write the Lagrangian for the pendulum.
2. Write the Euler-Lagrange equation.
3. Is this equation the same as you would derive from Newtonian formulation?

Homework 5. Due Friday, June 30, 10am.

Problem 1. *Rotating Pendulum*

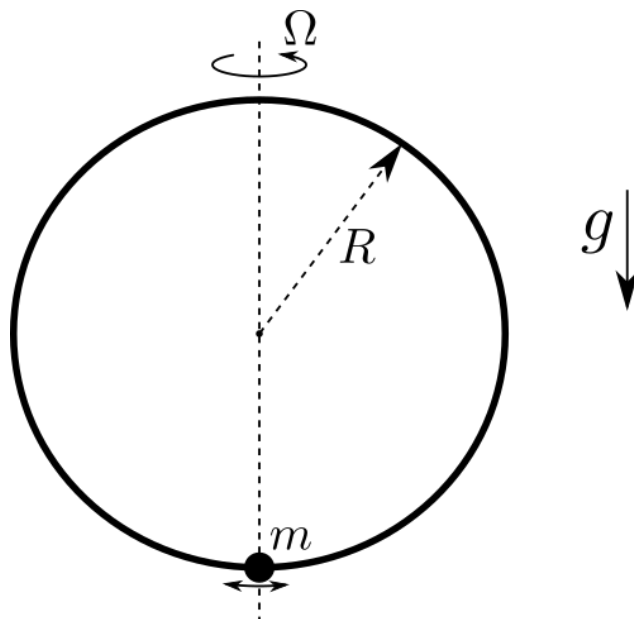
An ideal pendulum of mass m and length L is rotating around the vertical axis so that it makes a constant angle θ with the vertical line. Find the velocity of the mass m .



Problem 2. *Oscillations on Rotating Hoop*

A small bead of mass m can slide around the hoop of radius R without friction. The hoop is rotating around its diameter with constant angular velocity Ω around a vertical axis. Find the frequency of the very small oscillation of the bead close to its stable equilibrium position for Ω very close to Ω_c for both:

1. $\Omega < \Omega_c$.
2. $\Omega > \Omega_c$.



Homework 6. Due Friday, July 07, 10am.

Problem 1. *Captured light*

You want to capture light (dispersion relation $\omega = ck$, where ω is “angular” frequency, c is the speed of light, and k is a wave number $k = 2\pi/\lambda$, λ is the wavelength) into an ideal thin torus of radius R . Light of what frequency can you use?

Problem 2. *Strange wave*

A “strange wave” with a dispersion relation

$$\omega = \frac{\hbar}{2m}k^2,$$

where ω is the frequency, k is a wave number, and m and \hbar are known parameters is captured between two parallel ideal mirrors a distance L apart. What are allowed frequencies?

Problem 3. *Divergence*

Show, that for a vector field given by (k is some constant)

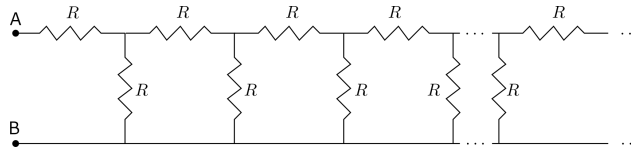
$$\vec{E} = k \frac{\vec{r}}{r^3}$$

everywhere except for the point $r = 0$, the following is true

$$\text{div}\vec{E} = 0.$$

Problem 4. *Semi-infinite ladder*

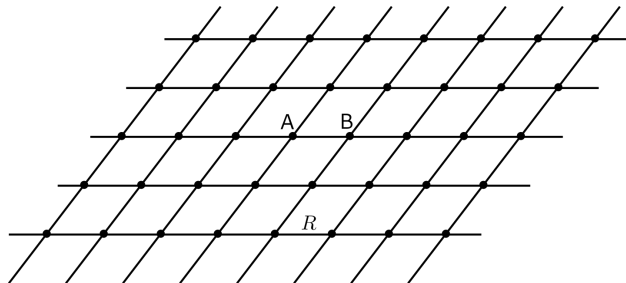
Find the resistance between the points A and B of a semi-infinite (it goes to infinity to the right) ladder of resistors show on the figure.



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Problem 5. *Lattice of resistors.*

An infinite square lattice is made of metal wire. The resistance of each link of the lattice is R . Find the resistance between the points A and B .



Homework 7. Due Friday, July 14, 10am.

Problem 1. *A ring of current*

A thin ring of radius R carries a current I . Find the magnetic field B at distance h right above the center of the ring.

Problem 2. *A ring of charge*

A thin ring of radius R is uniformly charged with the total positive charge Q .

1. Find the electric field at distance h right above the center of the ring.
2. Find the frequency of the small oscillations along the axis of the ring for a small negative charge q of mass m at the center of the ring.

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Problem 3. *Charge and loop*

A charge q with mass m has a velocity v far away. A small metallic loop of area A and resistance R is fixed and oriented such, that it and the vector \vec{v} are in the same plane. The impact parameter of the charge with respect to the loop is h . Assuming that h is much larger than the size of the loop, there is no gravity and no air resistance, find the velocity (magnitude and direction) of the charge after a very long time.

Homework 8. Due Friday, July 21, 10am.

Problem 1. *Vector potential*

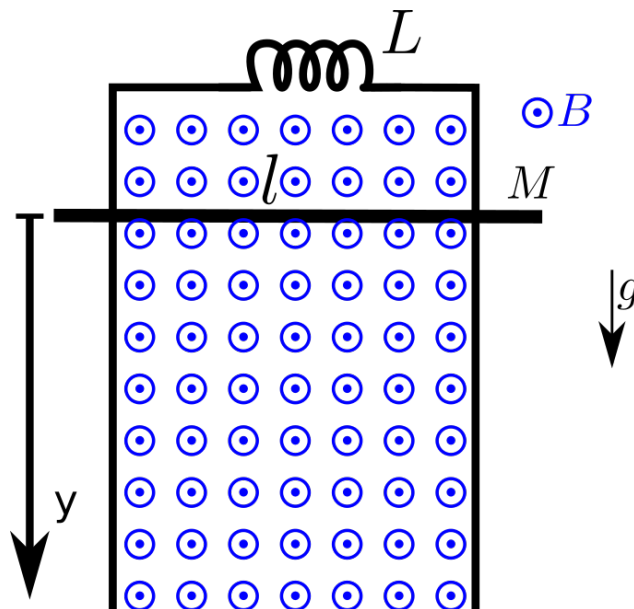
A magnetic field is described by a time independent vector potential $\vec{A}(x, y, z)$ such that $A_x = By$, $A_y = 0$, $A_z = 0$.

1. Find the circulation of the vector potential \vec{A} around a square of a side a in the $x - y$ plane using the definition of circulation of a vector field directly (without the use of the Stokes theorem.)
2. Find the circulation with the use of the Stokes theorem.
3. Find the magnetic field everywhere.
4. Find the magnetic field flux through the square.

Problem 2. *Vertical Rails with inductor*

Long enough vertical rails distance l from each other made of an ideal conductor are connected by an inductor L . A bar of mass M , also made of an ideal conductor, can slide along the rails without friction and without losing electrical contact with the rails. A uniform magnetic field B is horizontal and perpendicular to the plane of the rails. At some point the bar was released with zero initial velocity. Neglect all resistances. Initial current in the inductor is zero.

Find $y(t)$ – the height of the bar as a function of time if $y(0) = 0$.



Homework 9. Due Friday, July 28, 10am.

Problem 1. *Special relativity*

1. How fast must a meter stick be moving if its length is observed to be 0.5m?
2. A clock on a moving spacecraft runs 1s slower per day relative to an identical clock on Earth. What is the relative speed of the spacecraft?
3. A rod moves with velocity v along x direction. The rod makes an angle θ with the x direction in its own frame of reference. What angle does the rod make with the x direction for the stationary observer?

Problem 2. *Red light green light*

How fast (in m/s, and miles per hour) should you drive towards the street light in order for the red light to appear green to you?

Problem 3. *Change of mass*

Two lumps of clay of equal rest masses m travel with velocities v towards each other. After collision they stick together in a single lump.

1. What is the mass M of the resulting lump?
2. Show that your result converts to the expected one if $v \ll c$?

Problem 4. *Accelerator*

Protons in an accelerator are accelerated to an energy 400 times their rest energy.

1. What is the speed of these protons?
2. What would be the speed if the energy doubles (become 800 times their rest energy)?

Problem 5. *Compton scattering*

Compton used photons of wavelength 0.0711nm.

1. What is the energy of these photons?
 - (a) In Joules?
 - (b) In electron-Volts?
2. What is the wavelength of the photons scattered 180° (backscattering case)?
3. What is the energy of the back-scattered photons (in electron-Volts)?
4. What is the recoil energy of the electrons in this case (in electron-Volts)?

Homework 10. The last one. Due Friday, August 4, 10am.

You may use *Mathematica* or any other program to take the necessary integrals.

Problem 1. Wave function

An electron is described by a wave function:

$$\psi(x) = \begin{cases} 0 & \text{for } x < 0 \\ Ce^{-x/\lambda}(1 - e^{-x/\lambda}) & \text{for } x > 0 \end{cases},$$

where λ is a constant length, and C is the normalization constant.

1. Find C .
2. Where an electron is most likely to be found; that is, for what value of x is the probability for finding electron largest?
3. What is the average coordinate \bar{x} of the electron?
4. What is the standard deviation $\Delta x = \sqrt{\overline{x^2} - \bar{x}^2}$ of the electron position?
5. What is the average momentum \bar{p} of the electron?
6. What is the standard deviation $\Delta p = \sqrt{\overline{p^2} - \bar{p}^2}$ of the electron momentum?

Problem 2. Electron-positron annihilation

An electron (rest mass m_e) of total energy E makes a head-on collision with a positron, which is initially at rest. Positron is electron's antiparticle, it has the same mass as electron, but opposite charge. In collision the two particles annihilate each other and are replaced by two photons (γ rays) of equal energy, each traveling at equal angles θ with electron's direction of motion. Find

1. The energy ϵ_γ of each photon.
2. The momentum p_γ of each photon.
3. The angle θ .

