Homework 1. Due Friday, May 31.

Problem 1. *Cannon*
A cannon can shoot whatever cannons shoot at different angles to the horizon, but with the same initial velocity. At what angle does the cannon shoot to a maximal distance? What would be that angle on the moon?

Problem 2. *Colliding particles*
A point like particle collides with the identical particle which is initially at rest. The particles repel each other with the central force given by $\vec{F} = (\vec{r}_1 - \vec{r}_2)e^{-(\vec{r}_1-\vec{r}_2)^2/2l^2}$, where l is some constant and $\vec{r}_1$ and $\vec{r}_2$ are the positions of the particles. Find the angle between the particles' velocities after a long time after collision. Neglect the gravity.

Problem 3. *Rising Snake*
A snake of length L and linear mass density ρ rises from the table. It's head is moving straight up with the constant velocity v. What force does the snake exert on the table?

Problem 4. *Extra problem. Extra 5 points to the FINAL grade. Due by final exam.*
A boat floats in a lake. When the boat moves the force of resistance is proportional to the velocity of the boat. Initially the boat is at rest. A person walks from the stern to the bow of the boat. What will be the position of the boat long time after the person stopped moving?

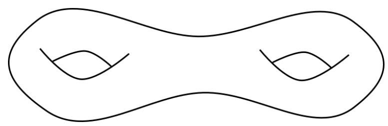
Homework 2. Due Friday, June 7.

Problem 1. *Sum of angles*

On a sphere a sum of all angles of a triangle is larger than π. What is the largest possible sum of all angles of a triangle on a sphere? What is it for a triangle on a hemisphere?

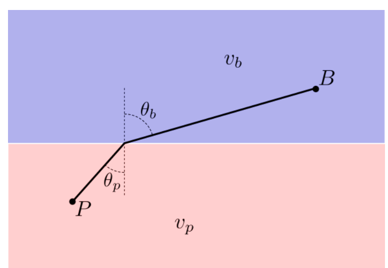
Problem 2. *Double torus*

Approximate a body shown on the figure by a polyhedral and compute $V + F - E$. Does it equal to $2 - 2g$? (Draw the polyhedral and write down V, F, and E.)



Problem 3. *Blue and pink*

A person can run with velocity v_b in the blue region and with velocity v_p in the pink region in the figure. What is the relation between θ_p and θ_b for the path that takes the person the <u>minimal time</u> to run from the point P to the point B?



Homework 3. Due Friday, June 14.

Problem 1. *Conservative forces*

- Which of the two $2D$ forces $\vec{F}^A(x,y)$ or $\vec{F}^B(x,y)$ given below is conservative? The x and y components of the force $\vec{F}^A(x,y)$ are given by

$$F_x^A(x,y) = -6x - y\cos(xy) - 14xy,$$
$$F_y^A(x,y) = -5 - x\cos(xy) - 7x^2$$

  and the x and y components of the force $\vec{F}^B(x,y)$ are given by

$$F_x^B(x,y) = -6x - y\cos(xy) - 12xy,$$
$$F_y^B(x,y) = -5 - x\cos(xy) - 7x^2.$$

- Write the potential $U(x,y)$ for the conservative one?

Problem 2. *Free fall*

Derive the formula $x(t) = x_0 + v_0 t + \frac{at^2}{2}$ for the $1D$ motion with constant acceleration a starting from the energy conservation law with potential energy $U(x) = -max$.

Problem 3. *May the fourth be with you*

A particle of mass m is moving in $1D$ in a potential $U(x) = kx^4$, where $k > 0$. The total energy of the particle $E > 0$.

- Find the period T of motion of the particle. (You do not need to take the last integral. Just write it.)
- How the period T depends on the total energy E?
- How the period of motion T depends on the total energy E if the potential energy is given by $U(x) = kx^2$?
- What will happen in both cases if $k < 0$?

Homework 4. Due Friday, June 21.

Problem 1. *A hole in the Earth*

1. A straight hole is drilled from the north pole to the south pole of the Earth. What time will it take for a ball dropped into the hole on the north pole to appear in the south pole? Neglect air resistance. Take the Earth to be a uniform sphere. Express your answer through the acceleration of free fall g and Earth radius R.

2. Compare the time you obtained in the part 1 to the time it take a satellite to go from the north pole to the south pole.

Problem 2. *Pendulum*

A simple pendulum of length l and mass m is oscillating in the Earth gravitational field. Using its angle ϕ as a coordinate.

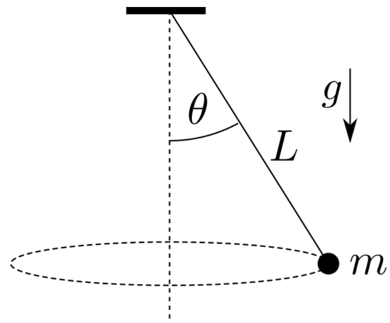
1. Write the Lagrangian for the pendulum.

2. Write the Euler-Lagrange equation.

3. Is this equation the same as you would derive from Newtonian formulation?

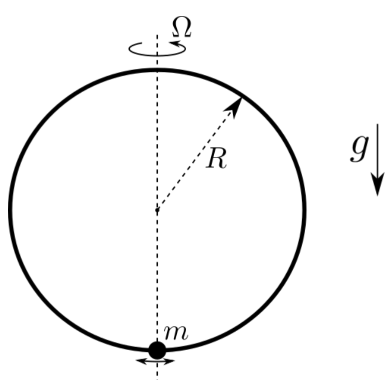
Homework 5. Due Friday, June 28.

Problem 1. *Rotating Pendulum*
An ideal pendulum of mass m and length L is rotating around the vertical axis so that it make a constant angle θ with the vertical line. Find the velocity of the mass m.



Problem 2. *Oscillations on Rotating Hoop*
A small bead of mass m can slide around the hoop of radius R without friction. The hoop is rotating around its diameter with constant angular velocity Ω around a vertical axis. Find the frequency of the very small oscillation of the bead close to its stable equilibrium position for Ω very close to Ω_c for both:

1. $\Omega < \Omega_c$.
2. $\Omega > \Omega_c$.



Homework 6. Due Monday, July 08.

Problem 1. *Captured light*
You want to capture light (dispersion relation $\omega = ck$, where ω is "angular" frequency, c is the speed of light, and k is a wave number $k = 2\pi/\lambda$, λ is the wavelength) into an ideal thin torus of radius R. Light of what frequency can you use?

Problem 2. *Strange wave*
A "strange wave" with a dispersion relation

$$\omega = \frac{\hbar}{2m} k^2,$$

where ω is the frequency, k is a wave number, and m and h are known parameters is captured between to parallel ideal mirrors a distance L apart. What are allowed frequencies?

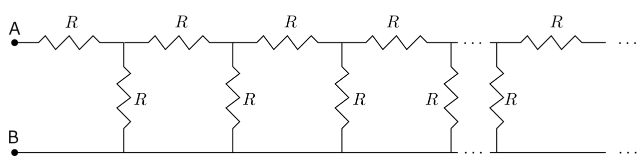
Problem 3. *Divergence*
Show, that for a vector field given by (k is some constant)

$$\vec{E} = k\frac{\vec{r}}{r^3}$$

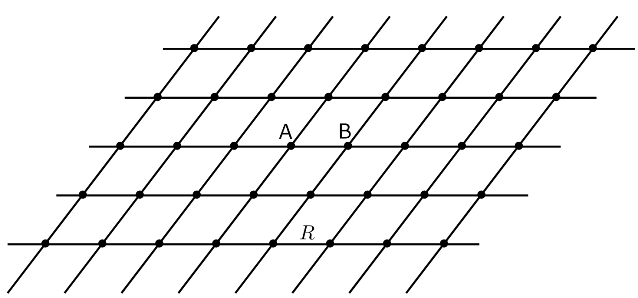
everywhere except for the point $r = 0$, the following is true

$$\mathrm{div}\vec{E} = 0.$$

Problem 4. *Semi-infinite ladder*
Find the resistance between the points A and B of a semi-infinite (it goes to infinity to the right) ladder of resistors show on the figure.



Problem 5. *Lattice of resistors.* *Extra 5 points to the FINAL grade. Due by final exam.*
An infinite square lattice is made of metal wire. The resistance of each link of the lattice is R. Find the resistance between the points A and B.



Homework 7. Due Friday, July 12.

Problem 1. *A ring of current*
A thin ring of radius R carries a current I. Find the magnetic field B at distance h right above the center of the ring.

Problem 2. *A ring of charge*
A thin ring of radius R is uniformly charged with the total positive charge Q.

1. Find the electric field at distance h right above the center of the ring.
2. Find the frequency of the small oscillations along the axis of the ring for a small negative charge q of mass m at the center of the ring.

Problem 3. *Extra problem. Extra 5 points to the FINAL grade.*
A charge q with mass m has a velocity v far away. A small metallic loop of area A and resistance R is fixed and oriented such, that it and the vector $\vec{v}$ are in the same plane. The impact parameter of the charge with respect to the loop is h. Assuming that $h \gg R$, there is not gravity and air resistance, find the velocity (magnitude and direction) of the charge after a very long time.

Homework 8. Due Friday, July 19.

Problem 1. *Vector potential*

A magnetic field is described by a time independent vector potential $\vec{A}(x, y, z)$ such that $A_x = By$, $A_y = 0$, $A_z = 0$.

1. Using the definition of circulation of a vector field, Find the circulation of the vector potential $\vec{A}$ around a square of a side a in the $x - y$ plane.

2. Find the magnetic field everywhere.

3. Find the magnetic field flux through the square.

Problem 2. *Special relativity*

1. How fast must a meter stick be moving if its length is observed to be 0.5m?

2. A clock on a moving spacecraft runs 1s slower per day relative to an identical clock on Earth. What is the relative speed of the spacecraft?

3. A rod moves with velocity v alone x direction. The rode makes an angle θ with the x direction in its own frame of reference. What angle does the rod make with the x direction for the stationary observer?

Problem 3. *Red light green light*

How fast (in m/s, and miles per hour) should you drive towards the street light in order for the red light to appeared green to you?

Homework 9. Due Friday, July 26.

Problem 1. *Change of mass*
Two lumps of clay of equal rest masses m travel with velocities v towards each other. After collision they stuck together in a single lump.

1. What is the mass M of the resulting lump?
2. Show that you result converts to the expected one if $v \ll c$?

Problem 2. *Accelerator*
Protons in an accelerator are accelerated to an energy 400 times their rest energy.

1. What is the speed of these protons?
2. What would be the speed if the energy doubles (become 800 times their rest energy)?

Problem 3. *Compton scattering*
Compton used photons of wavelength 0.0711nm.

1. What is the energy of these photons?
   (a) In Jouls?
   (b) In electron-Volts?
2. What is the wavelength of the photons scattered 180° (backscattering case)?
3. What is the energy of the back-scattered photons (in electron-Volts)?
4. What is the recoil energy of the electrons in this case (in electron-Volts)?

Homework 10. The last one. Due Friday, August 2.

You may use Mathematica or any other program to take the necessary integrals.

Problem 1. *Wave function*

An electron is described by a wave function:

$$\psi(x) = \begin{cases} 0 & \text{for } x < 0 \\ Ce^{-x/\lambda}(1 - e^{-x/\lambda}) & \text{for } x > 0 \end{cases},$$

where λ is a constant length, and C is the normalization constant.

1. Find C.
2. Where an electron is most likely to be found; that is, for what value of x is the probability for finding electron largest?
3. What is the average coordinate $\bar{x}$ of the electron?
4. What is the standard deviation $\Delta x = \sqrt{\bar{x^2} - \bar{x}^2}$ of the electron position?
5. What is the average momentum $\bar{p}$ of the electron?
6. What is the standard deviation $\Delta p = \sqrt{\bar{p^2} - \bar{p}^2}$ of the electron momentum?

Problem 2. *Semi-infinite potential well*

Consider a square well having an infinite wall at $x = 0$ and a wall of height U_0 at $x = L$. For the case $E < U_0$

1. Obtain solutions of the Scrödinger equation for $0 \le x \le L$ that satisfy the boundary condition at $x = 0$.
2. Obtain solutions of the Scrödinger equation for $L \le x$ that satisfy the boundary condition at $x \to \infty$.
3. Enforce the proper matching conditions at $x = L$ to find an equation for the allowed energies of the system.
4. Are there conditions for which no solution is possible for $E < U_0$?