Qualification Exam: Electromagnetism

Name: ________________________, QEID#62167059:

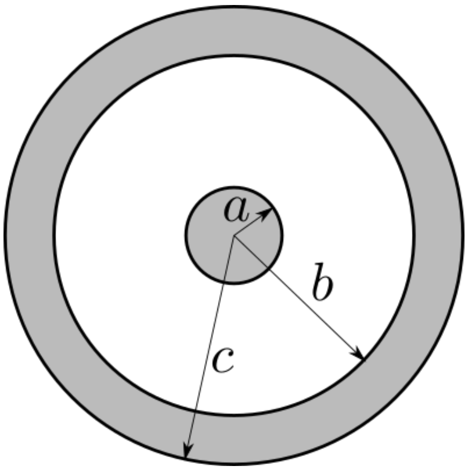
August, 2019

1 Undergraduate level

Problem 1. *1983-Fall-EM-U-1* ID:EM-U-2

Consider a coaxial cable carrying currents I in opposite directions in the two conductors.

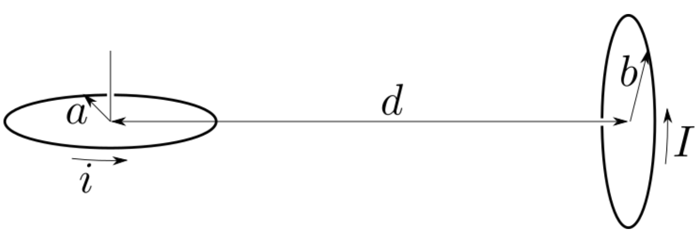
1. Find $\vec{B}(r)$ for all distances r from the symmetry axis.
2. Calculate the energy of the magnetic field per unit length of cable.
3. Calculate the self-inductance per unit length of the cable.



Problem 2. *1983-Fall-EM-U-2* ID:EM-U-5

Two current loops are very far apart ($d \gg a, b$) and oriented as shown.

1. In what directions are the force $\vec{F}$ and the torque $\vec{T}$ fe1t by the loop with radius b.
2. Compute the magnitudes of $\vec{F}$ and $\vec{T}$.



Problem 3. *1983-Fall-EM-U-3* ID:EM-U-8

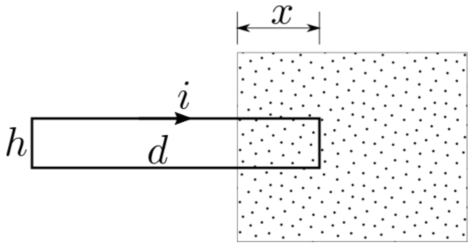
A conducting rectangular loop with sides of length h and d, resistance R, and self-inductance L is partially in a square region of uniform but time-varying magnetic field perpendicular to this region. The upward component of this field is given by $B_0 \sin(\omega t)$. The field outside the square region vanishes.

1. Assuming the loop to be fixed so that a length x of it is inside the field region, find the emf induced in the loop by the changing magnetic field.

2. Solve for $i(t)$, the clockwise current.

3. Compute the force on the loop. Find the time-averaged value of this force, $F_{\text{ave}}(x)$ to lowest non-vanishing order in $\omega L/R$.

4. Assume that the $F_{\text{ave}}(x)$ calculated in the previous part also applies if x varies slowly. How long does it take the loop to leave the field, if initially it is at rest and half of it is inside the field region?



Problem 4. *1983-Spring-EM-U-1* ID:EM-U-11

A straight metal wire of conductivity σ and cross-sectional area A carries a steady current I.

1. Determine the direction and magnitude of the Poynting vector at the surface of the wire.

2. Integrate the normal component of the Poynting vector over the surface of the wire for a segment of length L, and compare your result with the Joule heat produced in this segment.

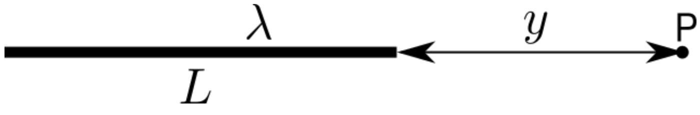
Problem 5. *1983-Spring-EM-U-2* ID:EM-U-14

A charge per unit length λ is distributed uniformly along a straight line segment of length L.

1. Determine the electrostatic potentia] (chosen to be zero at infinjty) at a point P a distance y from one end of the charged segment and in line with it (see the figure).

2. Use the result of the previous part to compute the component of the electric field at P in the y-direction (along the line).

3. Determine the component of the e1ectric field at P in a direction perpendicular to the straight line.



Problem 6. *1983-Spring-EM-U-3* ID:EM-U-17

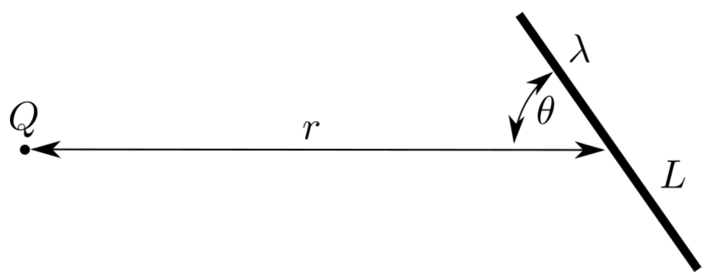
Given a magnetic field of cylindrical symmetry, i.e., one with a z-component $B_z = B(r)$, where r is the distance from the symmetry axis. An ion of charge q and mass m revolves in a circular orbit at distance R from the symmetry axis with angular velocity $\omega = qB(R)/m$. If the magnetic fieid is slowly increased in magnitude, show that the emf induced around the ion's orbit is such as to accelerate the ion. Show that in order for the ion to stay in the same orbit, the average increase in $B(r)$ over the surface enclosed by the orbit must be twice as large as the increase in $B(R)$.

Problem 7. *1984-Fall-EM-U-1* ID:EM-U-20

A line segment with uniform charge density λ and length L is free to rotate in a plane about its center which is fixed at a distance r from a fixed point charge Q. A line from Q to the center of the segment meets the segment at angle θ as shown. Assume $L \ll r$.

1. To leading non-vanishing order in L/r, compute the electrostatic energy of this configuration.

2. To leading non-vanishing order in L/r , calculate the torque acting on the segment about its center. For what θ is the system in equilibrium and is the equilibrium stable?



Problem 8. *1984-Fall-EM-U-2* ID:EM-U-23

A sphere of radius R is made of a linear permeable material, i.e. $\vec{B} = \mu\vec{H}$. It is placed in an external magnetic field $\vec{B} = B_0\vec{z}$. What are the fields $\vec{B}$, $\vec{H}$, and $\vec{M}$ everywhere in space? (Hint: $\nabla \times \vec{H} = 0$)

Problem 9. *1984-Fall-EM-U-3* ID:EM-U-26

Given a region of space in which the electric and magnetic fields are static and the electric field is parallel to the x axis:

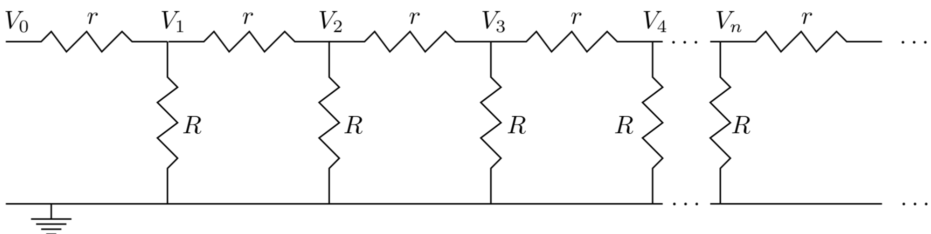
1. Prove that the electric field is independent of y and z.

2. If there is no charge in this region, prove that $\vec{E}$ is constant in this region.

Problem 10. *1984-Spring-EM-U-1* ID:EM-U-29

An infinite number of res1stors are connected in a ladder as shown:

1. In terms of r and R, what is the resistance between the point at potential V_0 and ground? Hint: adding an extra r and R in the pattern shown would not change this resistance.
2. Show that the potential V_i at the ith point shown is given by $V_i = V_0 c^i$. Calculate the ratio of r/R for which $c = 1/2$.



Problem 11. *1984-Spring-EM-U-2* ID:EM-U-32

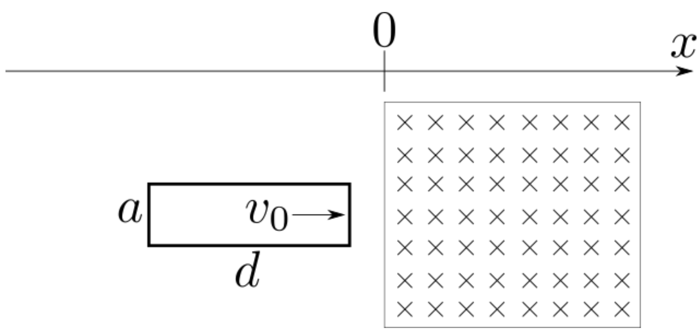
In three dimensions, with the potential taken to be zero at infinity, a charge Q lies at the origin and a charge $-q$ lies at ($x = a$, $y = 0$, $z = 0$).

1. Find the points on the x-axis a finite distance from the origin at which the electric potential vanishes.
2. Construct a sphere which passes through these points and whose center lies on the x-axis. Prove that the electric potential vanishes on this sphere.
3. This construction is useful in solving a set of physical problems. Explain how and why, and describe briefly and completely the set of problems.

Problem 12. *1984-Spring-EM-U-3* ID:EM-U-35

A rectangular loop of mass m, resistance R, and self-inductance L moves in the x direction on a frictionless surface. At time $t = 0$, the leading edge reaches $x = 0$ and the loop has speed v_0. In the region $x > 0$, there is a constant magnetic field B directed into the paper, as shown.

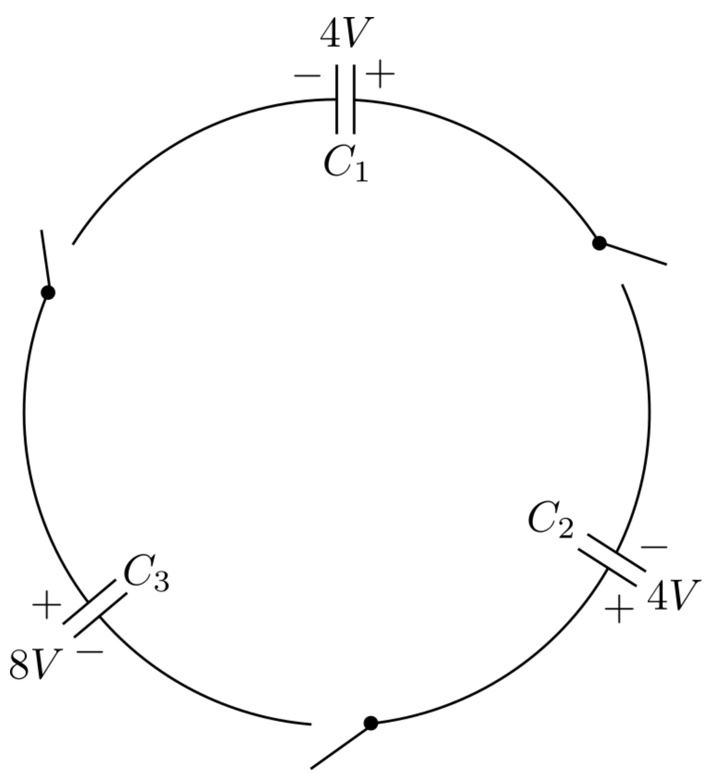
1. Find the induced emf and the equation of motion for the loop. Assume that for all times of interest, the trailing edge of the loop remains out of the magnetic field.
2. What relationship involving the parameters leads to motion which is critically damped?
3. In the case of critical damping, solve for the current $i(t)$ and the position of the leading edge, $x(t)$.



Problem 13. *1985-Fall-EM-U-1* ID:EM-U-38

Consider the circuit, consisting of 3 capacitors and 3 switches, shown below. $C_1 = 10\mu F$, $C_2 = 5\mu F$, and $C_3 = 2\mu F$. Each capacitor is charged to the potentiai and polarity indicated. Neglect fringing fields.

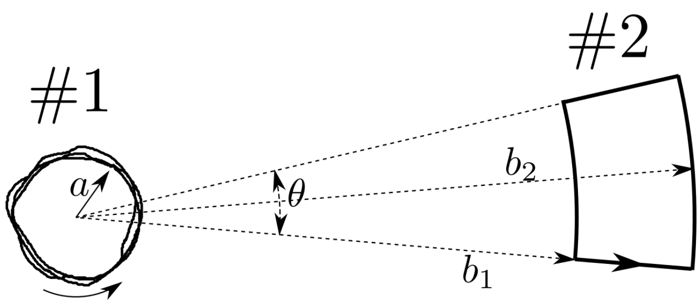
- How much energy is stored in the capacitors?
- The three switches are now closed. What is the new charge on each plate of each capacitor. Summarize your results by redrawing the circuit and indicating both the magnitude and the sign of the charge on each of the six plates.
- How much energy is stored in the capacitors now?
- Explain the difference between your answers to the last and the first parts.



Problem 14. *1985-Fall-EM-U-2* ID:EM-U-41

Consider the two coils shown below. Coil #1 consists of 3 turns of radius a. Coil #2 is a single turn, consisting of two circular sections with radii b_1 and b_2 connected by radial pieces. The angle θ between the two radial pieces is $\ll 1$. The coils are in the same plane, and the circular pieces are centered on the same point. Coil #1 has a resistance R. A current $I_2 = I_0 \sin(\omega t)$ flows in coil #2, in the direction indicated by the arrow. You may assume $a \ll b_1, b_2 \ll \lambda$.

1. What is the current induced in coil #1? Be sure to specify the direction, as well as the magnitude, of the induced current. The arrow indicates the "positive" direction.
2. Coil #1 is held rigidly 1n place. what is the force $\vec{F}(t)$ on coil #2?
3. What is the torque $\vec{N}(t)$ on coil #2?

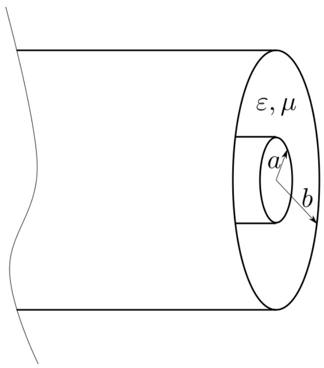


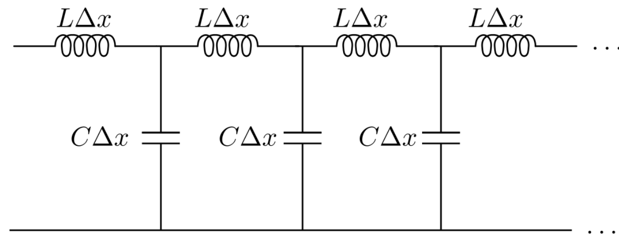
Problem 15. *1985-Fall-EM-U-3* ID:EM-U-44

A coaxial cable consists of two perfectly conducting, concentric cylinders. with radii a and b. The region between the cylinders is filled with a material with dielectric constant ε and permeability μ.

1. Calculate the capacitance per unit length C and the inductance per unit length L of the cable.
2. The characteristic impedance Z_0 of the cable is the ratio of the voltage between the cylinders to the total current flowing in either one. If we break the cab1e into short segments of length Δx and neglect fringing field effects, then each segment has a capacitance $C\Delta x$ and an inductance $L\Delta x$. Z_0 may be found by considering the cable to be an infinite chain of short segments in the limit $\Delta x \to 0$:

   Show that $Z_0 = \sqrt{L/C}$ independent of frequency. [Hint: Since the chain is infinitely long, the impedance is unchanged if one more segment is added at the beginning.]
3. Consider the phase difference $\Delta\phi$ across a short segment Δx of the cable as a function of frequency. Compare this to the change in phase $\Delta\phi$ of the same signal in a time Δt to show that the signal propagation velocity $v = 1/\sqrt{LC}$, also independent of frequency.



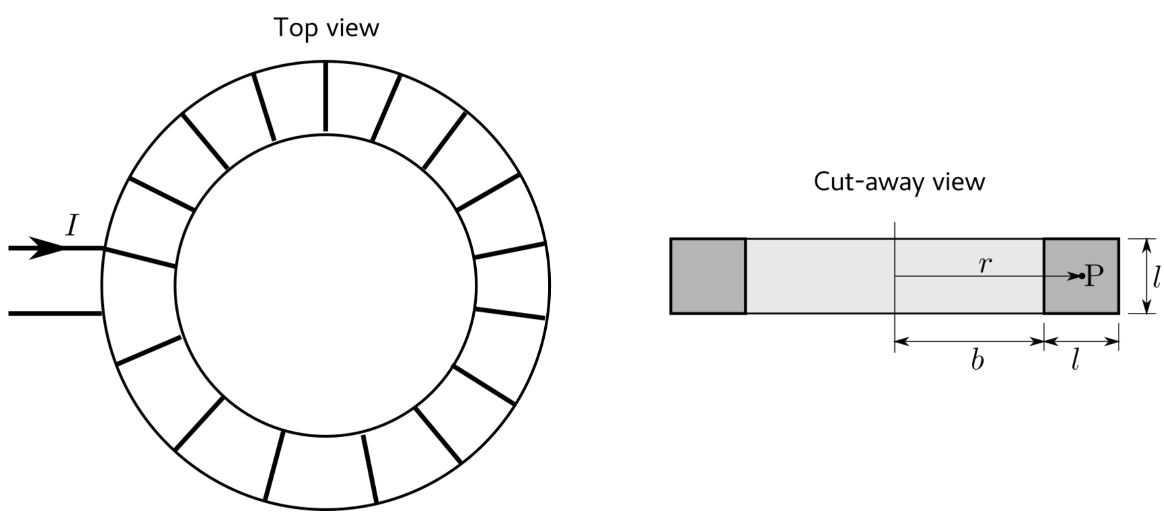


Problem 16. *1985-Spring-EM-U-1* ID:EM-U-47

A long tube with a square cross sectional area is formed into a toroid, as pictured below. The inner radius of the toroid is b and the outer radius is $b + l$, with l the height of the toroid. It is wrapped with N turns of wire, as drawn. A current I passes through the wire.

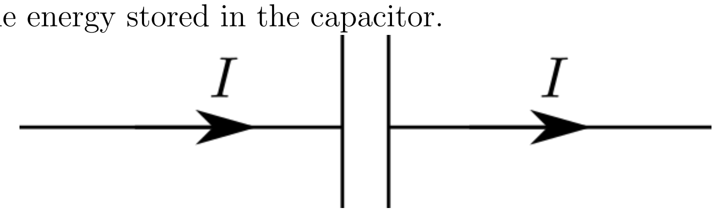
1. What is the magnetic field at the point P (shown in the picture) which is a distance r from the center of the toroid?

2. What is the self-inductance of this device?



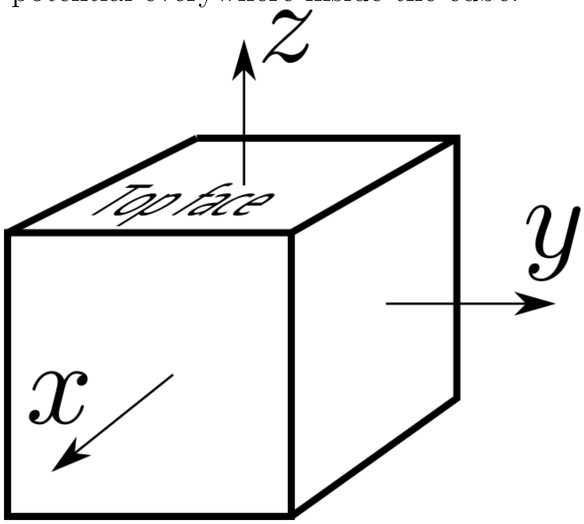
Problem 17. *1985-Spring-EM-U-2* ID:EM-U-50

Consider a circular capacitor with plates of radius a separated by a distance d. Assume $d \ll a$. The capacitor is being charged slowly by a current I. Show that the rate at which electromagnetic energy is entering the capacitor is equal to the rate at which the energy stored in the capacitor.



Problem 18. *1985-Spring-EM-U-3* ID:EM-U-53

A cube with sides of length b is centered at the origin. The sides and bottom are at a potential V_0. The top surface has the potential $\Phi = V_0 \cos(\pi v/b) \cos(\pi y/b)$. Find the potential everywhere inside the cube.



Problem 19. *1986-Spring-EM-U-1* ID:EM-U-56

A positive charge, with charge density ρ_0, is uniformly distributed throughout a large spherical volume of radius R.

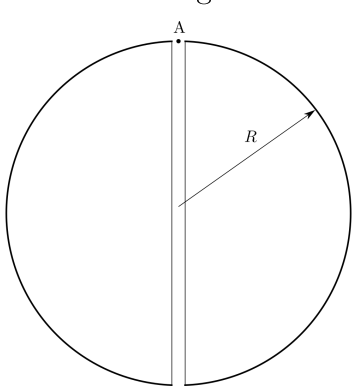
1. Calculate the electric field $E(r)$ at a point a distance r from the center of the distribution, both for $r < R$ and for $r > R$.

2. Calculate the electrostatic potential $\Phi(r)$ for $r < R$ and for $r > R$. Take $\Phi \to 0$ as $r \to \infty$.

   A very narrow tunnel is drilled along a diameter of the charge distribution, without otherwise altering it. A negative point charge $-q$ is released at the edge of the tunnel (point A in the figure). The charge in the spherical distribution is not able to move.

3. What is the velocity of the point charge when it reaches the center of the charge distribution?

4. How long does it take the point charge to return to point A?



Problem 20. *1986-Spring-EM-U-2* ID:EM-U-59

A hollow dielectric spherical shell with dielectric constant ϵ_1, inner radius a, and outer radius b is surrounded by another dielectric spherical shell – this one with dielectric constant ϵ_2, inner radius b, and outer radius c. Free charges Q_1, Q_2, and $Q_3 = -Q_1$ have accumulated on the surfaces at $r = a$, $r = b$, and $r = c$, respectively. These free charges are uniformly distributed over their respective surfaces.

1. What is the $\vec{E}$ field in the regions $a < r < b$, $b < r < c$, $c < r$?
2. What is the polarization charge density ρ_1 in the region $a < r < b$?
3. What is the polarization charge density σ_a on the surface at $r = a$?
4. What is the net polarization charge density σ_b on the surface at $r = b$?
5. What is the electrostatic force per unit area on a small patch of the free charge on the surface at $r = c$?

Problem 21. *1986-Spring-EM-U-3.jpg* ID:EM-U-62

A line of charge is located on the z axis from $z = -a$ to $z = a$. The charge density along this line is given by:

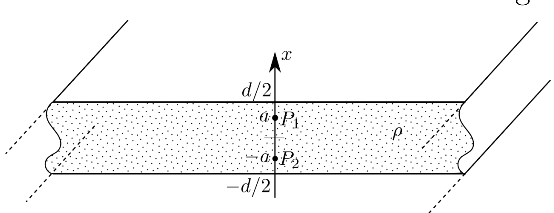
$$\lambda(z) = \frac{Q}{2a}\sin(\pi z/a).$$

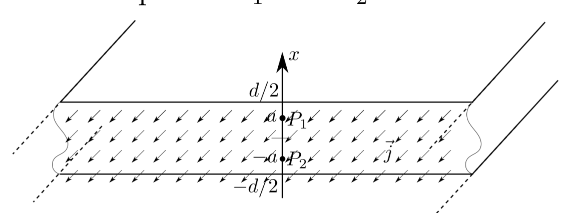
1. Find the electrostatic potential $\Phi(\vec{r})$ and the electric field $\vec{E}(\vec{r})$. Take $\Phi \to 0$ as $r \to \infty$. Give your answers in terms of one-dimensional integrals.
2. Approximate the potential $\Phi(\vec{r})$ in the region $|\vec{r}| \gg a$, and discuss the physical basis for your approximation.

Problem 22. *1987-Fall-EM-U-1* ID:EM-U-65

A slab of infinite length and infinite width has a thickness d. Point P_1 is a point inside the slab at $x = a$, and point P_2 is a point inside the slab at $x = -a$.

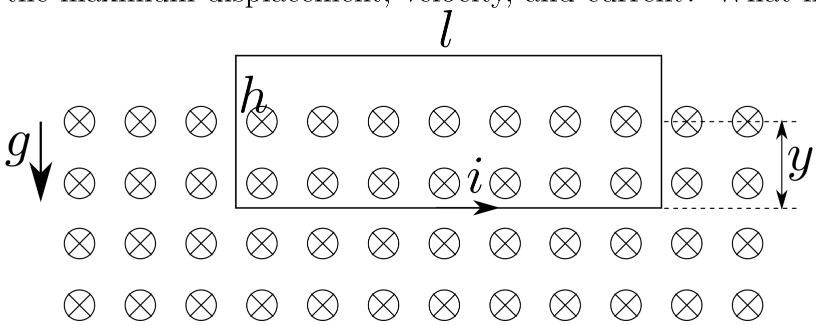
1. Consider the slab to be non-conducting with uniform charge per unit volume ρ, as shown on the figure. Use symmetry and Gauss's law to determine the direction and magnitude of $\vec{E}$ at points P_1 and P_2.
2. Consider the slab to be conducting and uncharged, but with a uniform current $\vec{j}$ directed out of the page as shown. Use symmetry and Ampere's law to determine the direction and magnitude of $\vec{B}$ at points P_1 and P_2.





Problem 23. *1987-Fall-EM-U-2* ID:EM-U-68

A superconducting wire, of inductance L, is held in place located partly in a uniform $\vec{B}$ field. It is then released, to be pulled downward by gravity. Letting a counter-clock-wise current be reckoned as positive, find an equation for the emf of this system. Also find the equation of motion for this system. Solve these equations subject to $y = 0$, $v_y = 0$, and $i = 0$ at $t = 0$. What is the frequency of oscillation ω? What are the maximum displacement, velocity, and current? What happens if h is too small?



Problem 24. *1987-Fall-EM-U-3* ID:EM-U-71

1. A point charge Q is located a distance y from the surface of an infinite plane conductor.

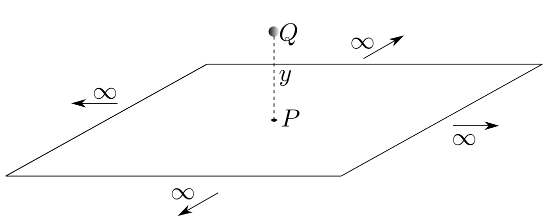
   (a) What is the magnitude and direction of the force on charge Q?

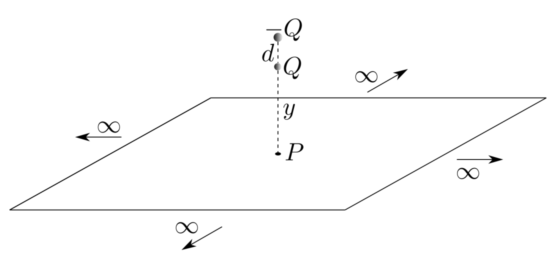
   (b) What is the surface charge density a on the surface of the conductor? (Give σ as a function of R, the distance to the point P on the conductor surface which is closet to Q).

2. A second charge $-Q$ is placed a fixed distance d above the first charge, forming a dipole as shown.

   (a) Find the net force and net torque on the dipole.

   (b) Find an approximate expression for the net force on the conducting plane due to the dipole, for $y \gg d$.





Problem 25. *1988-Fall-EM-U-1* ID:EM-U-74

An infinitely long, conducting cylinder of radius a at potential $\phi = 0$ is placed in an electric field, which approaches a uniform value $\vec{E}_0$ at large distances from the cylinder. The field is taken to be perpendicular to the axis of the cylinder.

1. Find the potential everywhere in space.

2. What is the electric field on the surface of the cylinder?

3. Calculate the charge per unit length along the cylinder.

Problem 26. *1988-Fall-EM-U-2* ID:EM-U-77

A particle with charge q is inside a dielectric sphere with radius a and dielectric constant ϵ.

1. a) If the particle is on the center of the sphere. Find $\vec{D}$, $\vec{E}$, and $\vec{P}$ everywhere in space. Be explicit about your choice of units. Find the polarization surface charge density on the surface $r = a$.

2. If the particle is moved off center a small distance δ in the $\hat{z}$ direction, calculate the force on it to leading order in δ/a.

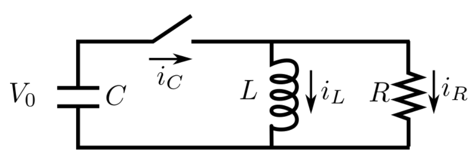
3. Discuss the stability of a central charge against small displacements.

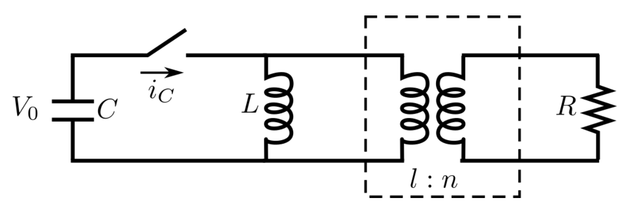
Problem 27. *1988-Fall-EM-U-3* ID:EM-U-80

A capacitance C, inductance L, and resistance R are connected in parallel as shown. With the switch open, a potential V_0 is placed across the capacitor.

1. The switch is now closed at time $t = 0$. Find the differential equation for the current i_L, through L and solve it for all times $t \geq 0$.

2. The resistance R is now replaced by an ideal transformer with the resistance R connected to the secondary coil. Assuming no power losses within the transformer and a primary to secondary coil ratio of $l : n$, find the current i_C through the switch when the capacitor held at potential V_0 is discharged by closing the switch. Do not concern yourself with the internal inductance within the dashed region.





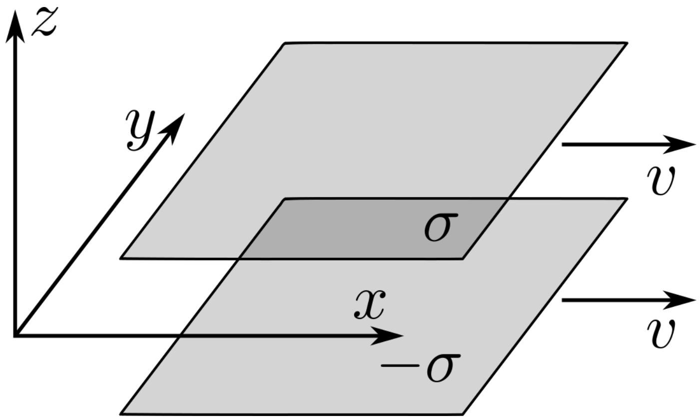
Problem 28. *1989-Fall-EM-U-1* ID:EM-U-83

Two infinite plane sheets of charge, parallel to the $x - y$ plane are located at $z = d/2$ and $z = -d/2$ respectively. In a reference frame F the sheets are observed to have charge densities σ and $-\sigma$ and each sheet moves with a velocity v in the $\hat{x}$ direction.

1. What are the electric and magnetic fields between the charge sheets according to an observer at rest in the frame F?

2. A second observer in a frame F' moves with a velocity v in the $\hat{x}$ direction according to the observer in F. What is the surface charge density on each sheet according to an observer in F'?

3. What are the electric and magnetic fields according to an observer in F'?



Problem 29. *1989-Fall-EM-U-2* ID:EM-U-86

A particle of charge Q moves along the z axis with a velocity $u < c$. The particle starts from $z = -\infty$ at time $t = -\infty$ such that it passes through the origin at $t = 0$.

1. Consider a circle of radius R lying in the $x - y$ plane. Find the electric flux through this circle at time t.

2. Using the time dependence of the result found in the previous part, find the magnetic field at an arbitrary point in the $x - y$ plane as a function of time.

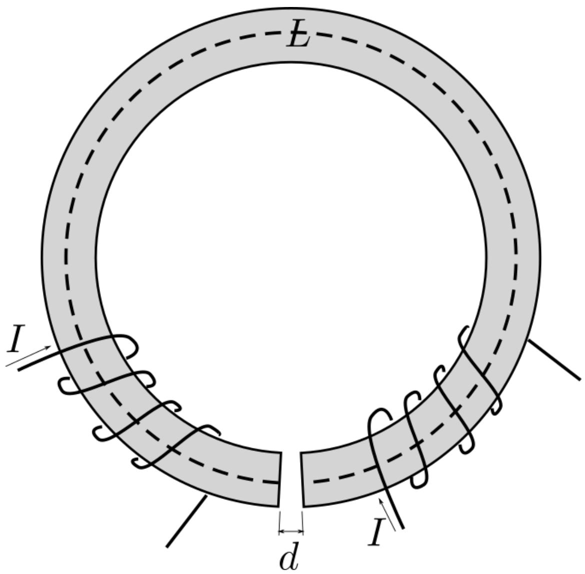
3. Make a sketch of the time dependence of the magnetic field found in the previous part. What is the maximum value of the field at a point (d, d, d) and at what time does the maximum occur?

Problem 30. *1989-Fall-EM-U-3* ID:EM-U-89

1. An infinite plane separates two magnetic materials with permeabilities μ_1 and μ_2. Calculate from Maxwell's equations the change in both the normal and tangential components of both $\vec{B}$ and $\vec{H}$ across the interface separating the two media.

2. Using the boundary conditions found in the previous part, estimate the magnetic field in the air gap of the following laboratory magnet. The path length through the iron is L and its permeability is μ. The small air gap is of length d and the magnet is energized by two coils of N turns each. A current I flows in each turn of the coils.

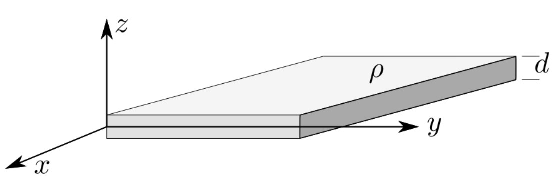
3. If $\mu/\mu_0 = 1000$, $L = 10$cm, $d = 0.5$cm. and $I = 2$amp, about how large is $|\vec{B}|$ in the air gap?

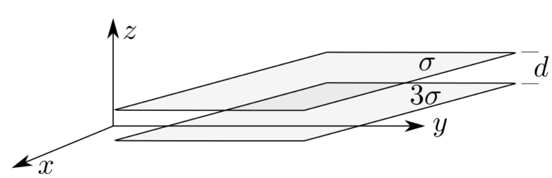


Problem 31. *1989-Spring-EM-U-1* ID:EM-U-92

Consider an infinite sheet of insulating material of finite thickness d, and dielectric constant $\kappa = 1$. Initially it has a uniform volume charge density, ρ_{free}, distributed throughout the slab.

1. Find the electric field produced by this charge distribution for any point in space.
2. Suppose this insulating sheet can be made a perfect conductor by raising its temperature. The charge in the previous part then redistributes itself in the conductor and comes to equilibrium. Find the electric field produced by this new charge distribution.
3. Consider now two very thin infinite sheets of charge, with charge per unit area as shown in the figure below. Find the electric field for all values of z.
4. Calculate the force per unit area that one sheet exerts on the other.





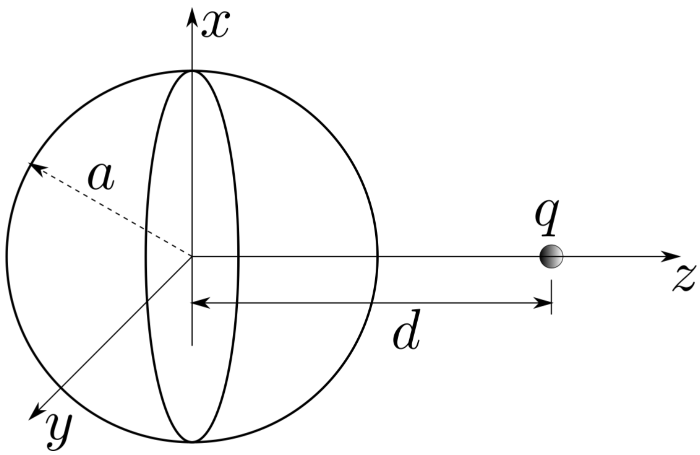
Problem 32. *1989-Spring-EM-U-2* ID:EM-U-95

You are given a grounded conducting sphere of radius a, centered on the origin. A point charge, q, is placed at the point $(0, 0, d)$, where $d > a$. Use the "Method of Images" to answer the following:

1. Find the $\vec{E}$-field for all points outside the conducting sphere.
2. Find the surface charge density on the sphere.

3. Suppose we make a very thin cut across the diameter of the sphere in the $x - y$ plane, dividing the sphere into two hemispheres. What is the electric force on the hemisphere closest to the point charge?

4. What limit does this force approach as $d \to a$?



Problem 33. *1989-Spring-EM-U-3* ID:EM-U-98

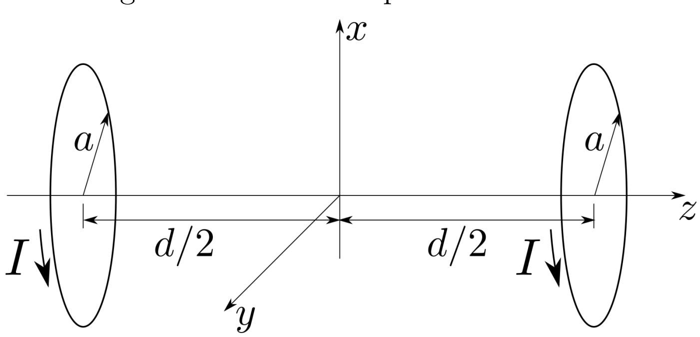
Consider two circular loops of wire, each of radius a, oriented parallel to the $x - y$ plane, with their centers located at $(0, 0, d/2)$ and $(0, 0, -d/2)$ as shown in the figure. Each loop carries a current I, in the same direction, as shown in the figure.

1. Find the magnetic field $\vec{B}(z)$, produced by the upper loop, for any point on the z axis. Make a rough sketch of $\vec{B}(z)$.

2. Find the total magnetic field $\vec{B}_{\text{tot}}$ produced by both loops for any point on the z axis. Make a rough sketch of $\vec{B}_{\text{tot}}$ for $d \sim 2a$.

3. This system of coils can be used to make a region of extremely uniform magnetic field, if d/a is suitably chosen. Expand the magnetic field along the z axis in a power series in z about the origin, and find the value of d which will make the field most uniform near the origin.

4. For a current $I = 10$A, $a = 0.5$m and $d = 0.5$m, find the value for $\vec{B}$ at the origin due to both loops.



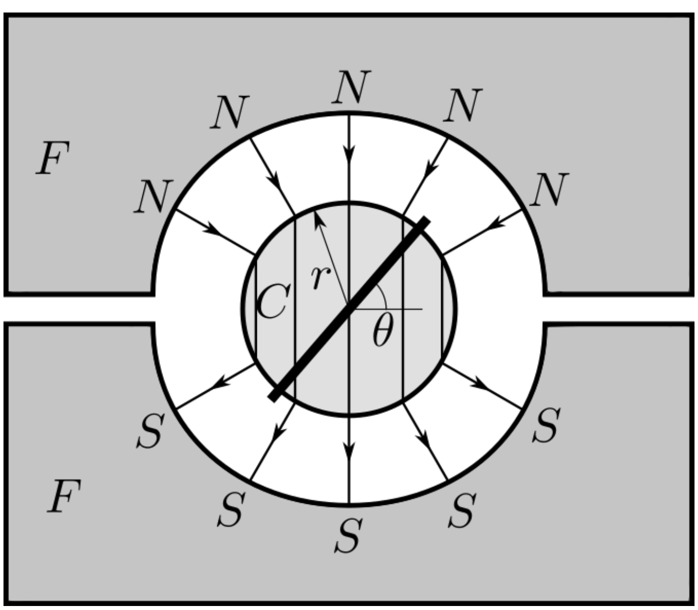
Problem 34. *1990-Fall-EM-U-1* ID:EM-U-101

The figure shows a highly simplified motor. Its fixed parts are a permanent magnet F and a "soft" iron core C, which produce a magnetic field $\vec{B}$ (where $B = |\vec{B}|$ is nearly independent of angle, and constant in time) that leaves the N end of F and points radially inward toward C, then goes vertically downward within C, and finally points radially outward from C until it enters the S pole of F.

Its moving put is an armature of moment of inertia I , radius r (shown) and length l (normal to the page), on which is wrapped a single rectangular loop of insulated wire, of resistance R and self-inductance L. The armature, which surrounds C, can rotate without friction about its axis (also normal to the page). The loop is connected to an emf $\mathcal{E}$ by a pair of sliding contacts (known as a commutator) which reverses the direction of the current i whenever the plane of the loop crosses the field-magnet gap (i.e., whenever $\theta = n\pi$, where n is an integer). At $t = 0$ the armature is at rest with θ slightly larger than zero.

The circuit equation for this system is $\mathcal{E} + \mathcal{E}_{\mathrm{b}} + L di/dt = iR$, and the torque equation is $\Gamma = I d\omega/dt$.

1. Derive an expression for the back emf $\mathcal{E}_{\mathrm{b}}$.
2. Derive an expression for the torque Γ acting on the loop of wire.
3. Find the current at times short enough that the armature has not yet come into rotation, but long enough that the self-inductance can be neglected.
4. Find the current when the armature reaches its final operating angular velocity ω_f at long times, and find ω_f.
5. Neglecting the self-inductance, solve for the angular velocity ω as a function of time t. Give the characteristic time for the motor to come into operation.



Problem 35. *1990-Fall-EM-U-2* ID:EM-U-104

Consider a spherically symmetric charge density $\rho = A(1 - r/a)$ for $r < a$, and zero for $r > a$. It is enclosed in a concentric, grounded, sphere of radius $b > a$.

1. Find the electric field for $r < a$.
2. Find the electric field for $a < r < b$.
3. Find the potential for $r < a$.
4. Find the potential for $a < a < b$.

Problem 36. *1990-Fall-EM-U-3* ID:EM-U-107

A conducting cylinder of radius a and net charge λ per unit length is placed at the center of a larger (non—conducting) hollow cylinder of radius $b > a$. ρ and ϕ are the polar coordinates in the plane perpendicular to the cylinders with the origin on the cylinders' axes. The potential on the surface of the outer cylinder is $\Phi(b, \phi) = A\cos\phi + B$. The potential at infinity is a constant, Φ_∞.

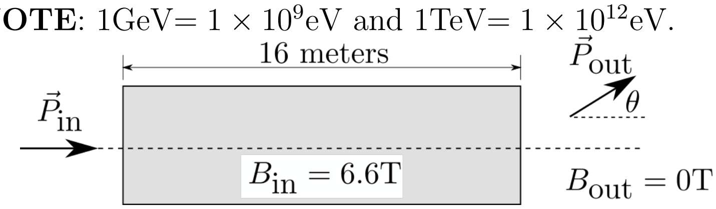
1. Write down the form that the potential must take for $\rho < a$, $a < \rho < b$, and for $b < \rho$. Do not try to evaluate any non-zero constants.
2. Matching boundary conditions, find the potential in each of these regions.
3. What is the net charge per unit length on the outer cylinder?

Problem 37. *1990-Spring-EM-U-1* ID:EM-U-110

A proton of mass .938GeV and total energy of 20TeV enters a magnet and is deflected as shown in the figure above. The magnet is 16 meters long and the B-field is uniform With $B = 6.6$ Tesla inside this volume and $B = 0$ outside the magnet(assume that there are no fringing fields).

1. In which direction must the field point?
2. Calculate the deflection angle, θ, for the proton as it passes through this magnet.
3. Estimate how many such magnets arranged in a circle would be required to trap the proton beam into circular orbit. What is the circumference of such a circle of magnets?

NOTE: 1GeV$= 1 \times 10^9$eV and 1TeV$= 1 \times 10^{12}$eV.



Problem 38. *1990-Spring-EM-U-2* ID:EM-U-113

Consider an infinite sheet at $z = 0$ that can carry a surface current, $\mathbf{K}$, in the $x - y$ plane. Consider that $\mathbf{K} = \int \mathbf{J} dz$ over the thickness of the sheet with $\mathbf{J}$ uniform.

1. Show that $\hat{z} \times (\mathbf{B}_2 - \mathbf{B}_1) = \frac{4\pi}{c}\mathbf{K}$, where $\mathbf{B}_2$ ($\mathbf{B}_1$) is the field for $z > 0$ ($z < 0$).
2. Let a magnetic monopole g be placed at a height h above the sheet, which is superconducting (i.e. $\mathbf{B}_1 = 0$ for $z < 0$). Find the field $\mathbf{B}$ for $z > 0$. [Hint: Use the method of images. Draw the field lines for this configuration.]
3. Find the force repelling the monopole from the superconductor.
4. Find the surface current density $\mathbf{K}$ that actually supports the monopole. Sketch the surface current distribution as seen from the positive z-direction looking down.

Problem 39. *1990-Spring-EM-U-3* ID:EM-U-116

An uncharged uniform dielectric sphere with dielectric constant, ε, and radius a is placed in an axially symmetric external field. Along the z-axis the polarization vector, $\mathbf{P}$, has a component of the form,

$$P_z = -\left(\frac{q}{a^3}\right)(a - z).$$

1. Find the values of $\mathbf{P}$, $\mathbf{D}$, and $\mathbf{E}$ inside the sphere.
2. Calculate the polarization surface charge density on the surface of the sphere.

Problem 40. *1991-Fall-EM-U-1* ID:EM-U-119

A changing magnetic field induces a uniform azimuthal current in a thin-walled cylindrical conductor of length l, average radius a, wall-thickness t, and resistivity ρ ($l \gg a \gg t$). (This might be done, for example, by having polarized, plane electromagnetic waves with $\mathbf{B}$-vector parallel to the axis of the cylinder incident upon it.)

1. Determine the resistance R to the flow of this current.
2. Determine (approximately) the self inductance L of the cylinder, by considering the related problem of a very long cylindrical solenoid of radius a, wound with n turns per unit length, each turn carrying current i:
   (a) Describe qualitatively the B-field produced by the solenoid for $r < a$ and for $r > a$.
   (b) Use Ampere's Law to determine B within the solenoid ($r < a$).
   (c) Determine the self-inductance L of the length l of the solenoid.
   (d) Reinterpret the result of the previous subpart to find L for the thin-walled conductor described in the first sentence.

Problem 41. *1991-Fall-EM-U-2* ID:EM-U-122

1. A house is located 200m south of a straight east-west road. A television set in the house receives signals from a distant transmitter, also situated south of the road, operating at a frequency of 60MHz. A bus, traveling west along the road, causes the received signal to fluctuate in intensity, the rate of fluctuation being 2Hz when the bus is opposite the house, and the rate (temporarily) reducing to zero when it is 400m further along the road. Find the speed of the bus and the direction of the transmitter. The speed of light is 3×10^8m/sec. (Neglect Doppler effect due to the small velocity of the bus.)

2. A highway patrol officer is clocking the speed of passing cars further down the road. The patrol car is located 15m from the side of the road. The central axis of its radar beam makes an angle of 20° with the road. If the transmitting antenna has a horizontal width of 0.2m and is operating at a wavelength of 3cm, over what distance along the road can it detect a passing car within the central diffraction peak?

Problem 42. *1991-Fall-EM-U-3* ID:EM-U-125

1. (a) In otherwise empty space, a neutral point dipole $\mathbf{p}$ is located at $\mathbf{r}_0 = z_0\hat{z}$. Find the electric potential for any $\mathbf{r} \neq \mathbf{r}_0$, expressing your answer in terms of the vectors $\mathbf{p}$, $\mathbf{r}$, $\mathbf{r}_0$ without breaking them up into components. Find the corresponding $\mathbf{E}$ field, expressing it in terms of the same vectors.

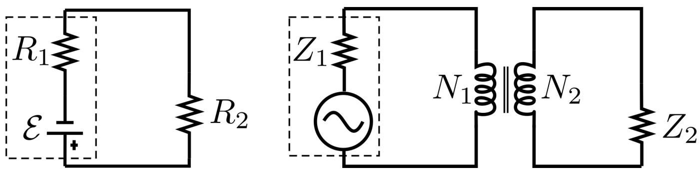
2. Taking $z_0 > 0$ in the previous part, now let all of the region with $z < 0$ be filled with grounded conductor. Show that for $z > 0$, the electric potential can be written as the sum of the potential of the previous part plus a potential corresponding to a dipole $\mathbf{p}'$ located at $\mathbf{r}'$. Find the relationships between the x, y, z components of $\mathbf{p}$ and $\mathbf{p}'$ and between $\mathbf{r}_0$ and $\mathbf{r}'_0$. Express the total potential in terms of the vectors $\mathbf{p}$, $\mathbf{p}'$, $\mathbf{r}$, $\mathbf{r}_0$, $\mathbf{r}'_0$. Express the $\mathbf{E}$ field in terms of the same vectors.

3. What condition(s) must the ρ and z components of $\mathbf{E}$ satisfy at the plane $z = 0$?

Problem 43. *1991-Spring-EM-U-1* ID:EM-U-128

1. A battery of emf $\mathcal{E}$ and internal resistance R_1 is connected in series with a load resistance R_2. Derive the relationship between R_1 and R_2 which gives optimum (i.e., maximum) transmission of power to R_2, and determine this maximum power.

2. An audio amplifier of output impedance Z_1 is connected to a loud speaker of impedance Z_2 via an ideal (i.e., lossless) transformer. Assuming that the phase angle is zero (i.e., that $Z_1 = R_1$ and $Z_2 = R_2$), derive a relationship for the turns-ratio N_2/N_1 of the transformer which gives optimum transmission of power to the speaker.

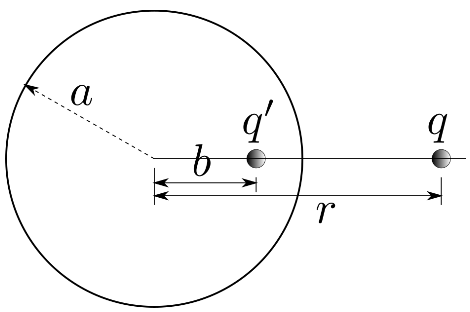


Problem 44. *1991-Spring-EM-U-2* ID:EM-U-131

1. Calculate the force on a charge q, at a distance r from an infinite conducting plane held at zero potential, by using the method of images.

2. Determine the value of the charge q' necessary to make the surface $r = a$ in the diagram a zero—potential surface, given that q' is located as shown, with $b = a^2/r$.

3. Hence calculate the force on a charge q located at a distance r from the origin of a grounded conducting sphere of radius a.



Problem 45. *1991-Spring-EM-U-3* ID:EM-U-134

Two circular conducting plates of radius a form a capacitor. The region between the plates is filled with a linear, homogeneous, isotropic material of permittivity ε, permeability μ, and conductivity σ. At time $t = 0$, the upper plate holds charge Q, the lower $-Q$. (Neglect edge effects.)

1. Express the current density $\vec{J}$ in the region between the plates in terms of the charge on the plates.

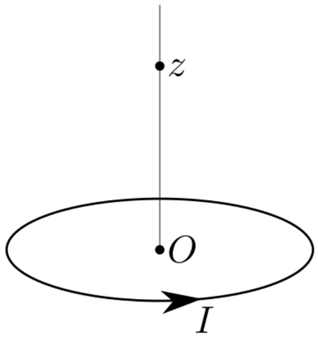
2. What is the charge on the plate as a function of time?

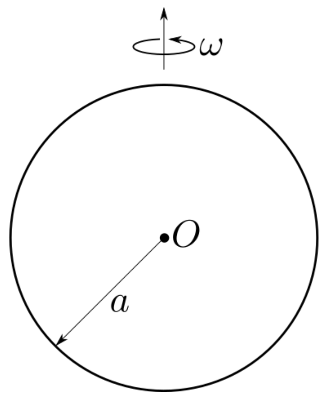
3. What are the fields $\vec{B}$ and $\vec{H}$ in the region between the plates?

Problem 46. *1992-Fall-EM-U-1* ID:EM-U-137

1. Calculate the magnetic field $\vec{B}$ on the axis of a ring carrying a current I, as shown.

2. A thin spherical shell of radius a has a charge Q uniformly distributed over its surface. It rotates about a diameter with angular frequency ω. Calculate the magnetic field at the center.





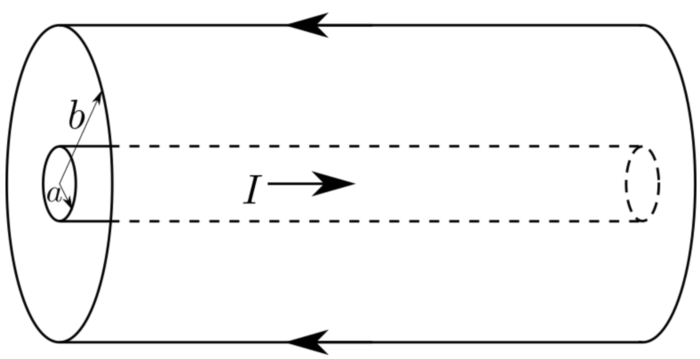
Problem 47. *1992-Fall-EM-U-2* ID:EM-U-140

A current I flows down a long thin wire of radius a and back along a thin, coaxial cylindrical shell of radius b. (See figure below.)

1. Calculate the energy per unit length stored in the cable in the region between and excluding the conductors.

2. Calculate the self inductance per unit length of the cable. (Ignore end effects and any contribution due to fields in the conductors.)

3. If the current I is made to oscillate according to the function $I = I_0 \cos(\omega t)$, find the electric field $\vec{E}$ in the region between the conductors as a function of r — the distance from the axis — to within an additive constant.



Problem 48. *1992-Fall-EM-U-3* ID:EM-U-143

In the frame S, two parallel infinite planes are at rest with surface charge density $+\sigma_0$ on one plate and $-\sigma_0$ on the other. Let the plane with positive charge be defined by the surface $z = a$ and the other by $z = -a$. In this frame, a particle with charge q travels along the x axis between the planes with a velocity $v_0\hat{x}$.

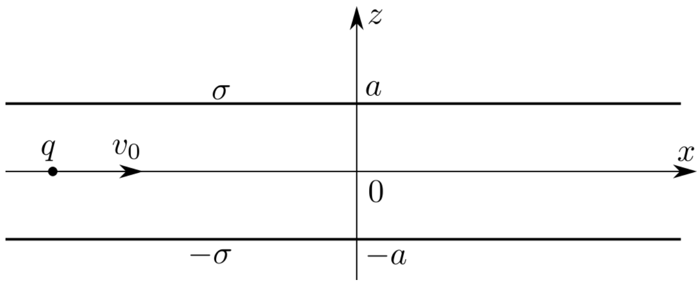
Consider frame S' in which the parallel planes have a velocity $c\hat{x}/2$ with c denoting the speed of light.

In terms of these given variables;

1. calculate the fields $\vec{E}'$ and $\vec{B}'$ "seen" by the particle in the frame S';

2. calculate the velocity of the particle in the frame S';

3. calculate the total electromagnetic force $\vec{F}'$ acting on the particle in the frame S'.

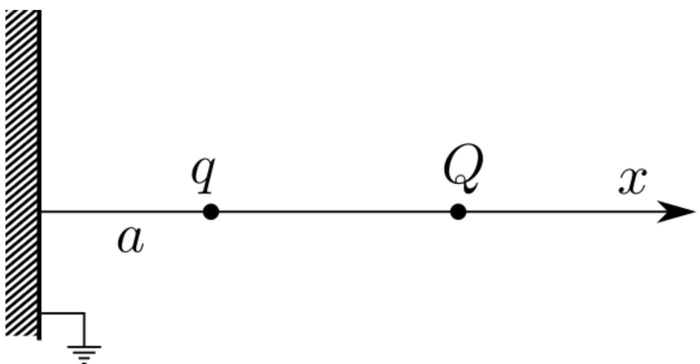


Problem 49. *1992-Spring-EM-U-1* ID:EM-U-146

A charge q is fixed at a distance a from an infinite, conducting, grounded plane. A second charge, Q, is constrained to move along the x axis as shown in the diagram.

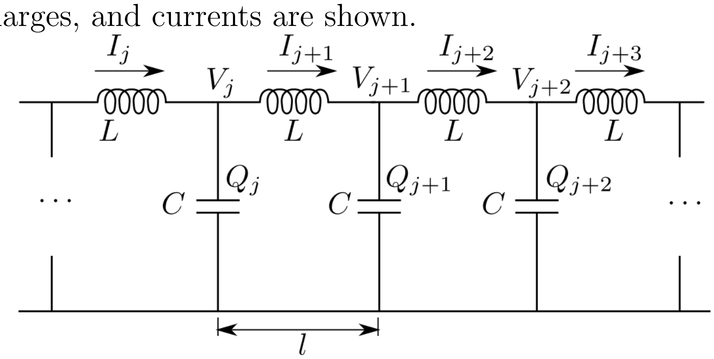
1. Find the value that the charge Q must have in order that it can rest in equilibrium at $x = 2a$.

2. Assuming that Q has a mass m, find the period T of small oscillations around the equilibrium position $x = 2a$.



Problem 50. *1992-Spring-EM-U-2* ID:EM-U-149

Consider an infinitely long transmission line which consists of lumped circuit elements L and C as shown. Find the dispersion relation (i.e., ω versus $k = 2\pi/\lambda$) for periodic waves traveling down this line (i.e., for solutions $I_j(t)$, $V_j(t)$, or $Q_j(t)$ which are periodic in j and t as in a traveling wave). What is the cutoff frequency? Voltages, charges, and currents are shown.



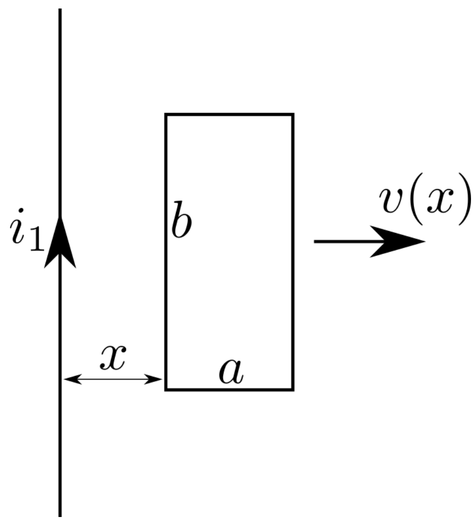
Problem 51. *1992-Spring-EM-U-3* ID:EM-U-152

Consider a long, straight wire carrying constant current i_1. A rectangular loop is positioned as shown, so that the wire and the loop are in the same plane. This loop has total resistance R, height b, width a, and zero mass.

1. Find the velocity required to maintain a constant current i_2 in the loop, as a function of x. (x is the distance from the wire to the loop, as shown.)

2. Find the external force, as a function of x, required to cause this velocity.

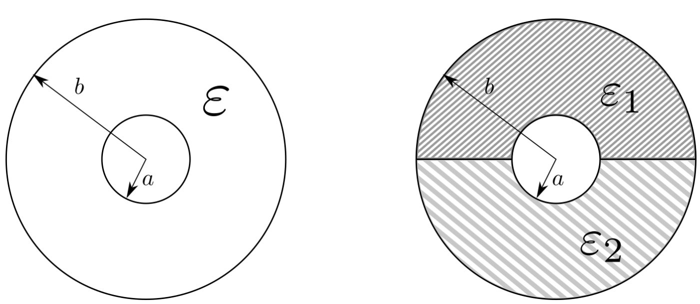
3. Find the mechanical power supplied by the force, and compare to the dissipation in the resistive loop.



Problem 52. *1993-Fall-EM-U-1* ID:EM-U-155

1. Find the capacitance of a cylindrical coaxial capacitor of length L, where the inner conductor has radius a and the outer conductor has radius $b \ll L$. The space between the conductors is filled with a medium with dielectric constant ε.

2. Suppose now that the space is filled instead with two different media, with dielectric constants ε_1 and ε_2, each occupying half the volume, as shown in the diagram. What is the capacitance now?



Problem 53. *1993-Fall-EM-U-2* ID:EM-U-158

A straight circular cylindrical metal wire of uniform conductivity σ and cross-sectional area A carries a steady current I.

1. Determine the direction and magnitude of the Poynting vector at the surface of the wire.

2. Integrate the normal component of the Poynting vector over the surface of the wire for a segment of length L.

3. Compare your answer with the Joule heat produced in this segment and interpret this result.

Problem 54. *1993-Fall-EM-U-3* ID:EM-U-161

A point charge q is placed a distance b from the center of a grounded conducting sphere of radius a ($a < b$). Find the force between the point charge and the sphere.

Problem 55. *1993-Spring-EM-U-1* ID:EM-U-164

A point charge Q is located a distance D from an infinite grounded conducting plane. Assume the plane is the $x - y$ plane and the charge is located on the z axis.

1. Find the force on the point charge.

2. Find the surface charge density induced on the plane.

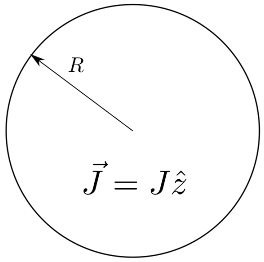
3. Find the total charge induced on the plane by integrating the result of the previous part.

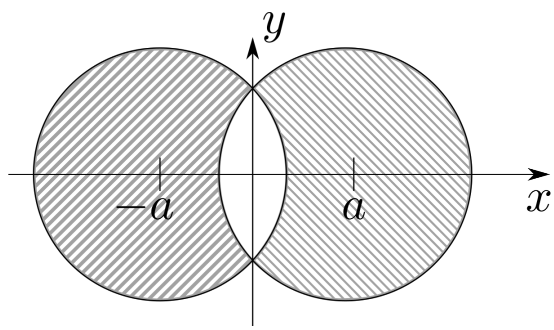
4. Find the total energy of the configuration.

Problem 56. *1993-Spring-EM-U-2* ID:EM-U-167

1. A conducting cylinder of radius R is carrying a uniform current density J . Let the axis of the cylinder coincide with the z axis, so $\vec{J} = J\hat{z}$. Find the magnetic field both inside and outside the conductor.

2. Consider a current configuration constructed of two interpenetrating cylinders, with the material in the overlap region removed as shown. In the given coordinate system, the conductor on the left has a uniform current density $\vec{J}_{\text{left}} = J\hat{z}$ along its axis. The one on the right has the same current distribution, but in the opposite direction ($\vec{J}_{\text{right}} = -J\hat{z}$). Find the magnetic field in the overlap region in terms of the given variables and coordinate system.

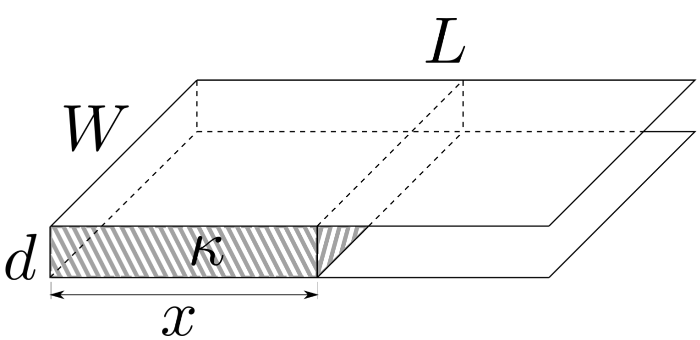




Problem 57. *1993-Spring-EM-U-3* ID:EM-U-170

The space between two parallel plates of width W, length L and separation at is filled partially by vacuum and partially by a slab with dielectric constant κ as shown ($\varepsilon = \kappa\varepsilon_0$). (IGNORE FRINGE EFFECTS in this problem.)

1. Find $\vec{E}$, $\vec{D}$ and $\vec{P}$ (polarization) in BOTH
   (a) the vacuum region and the dielectric region assuming the plates are conducting and there is a constant potential V across the plates. (The top plate is positively charged relative to the bottom plate.)
2. Find the force on the dielectric.
3. Find the same quantities as in the first part, but for the case where the plates have FIXED surface charge densities $+\sigma_0$ and $-\sigma_0$ on the top and bottom plates respectively.



Problem 58. *1994-Fall-EM-U-1* ID:EM-U-173

A solid metal sphere of radius R has charge $+2Q$. A thin hollow spherical conducting shell of radius $3R$ is placed concentric with the first sphere, and has net charge $-Q$.

1. Find the expression for the magnitude of the electric field between the spheres at a distance r from the center of the inner sphere ($R \leq r \leq 3R$).
2. Calculate the electric potential for all r. (Take $\phi = 0$ at $r = \infty$.)
3. What would be the final distribution of the electric charge if the two spheres were joined by a wire?

Problem 59. *1994-Fall-EM-U-2* ID:EM-U-176

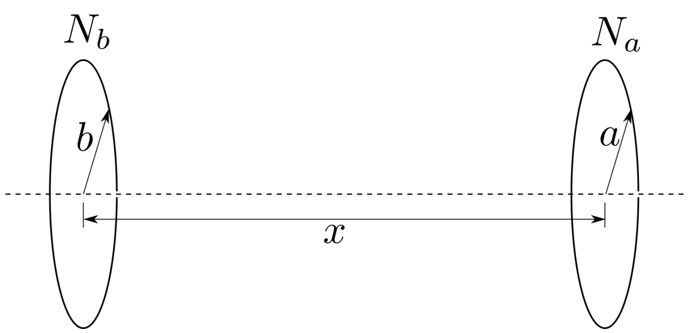
Consider the figure below:

1. (a) The currents in the coils are downward on the near side, present a simple argument that tells whether they attract or repel.

2. The currents in the coils are initially zero. A battery (not shown) causes current in the left loop to go downward on the near side. Present simple arguments that give the direction of the current induced in the right coil, whether the coils attract or repel, and whether the radial force on the coil on the right due to the coil on the left tends to compress it or expand it.

3. For $x \gg a, b$ compute the mutual inductance of the two coils. Recall, the Biot-Savart Law is $\vec{B} = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \hat{r}}{r^2}$.

4. If $N_b = 100$, $N_a = 40$, $x = 10$cm, $b = 2$cm, $a = 1$cm, and $dI_a/dt = 4 \times 10^6$A/sec, find the emf induced in the right coil.



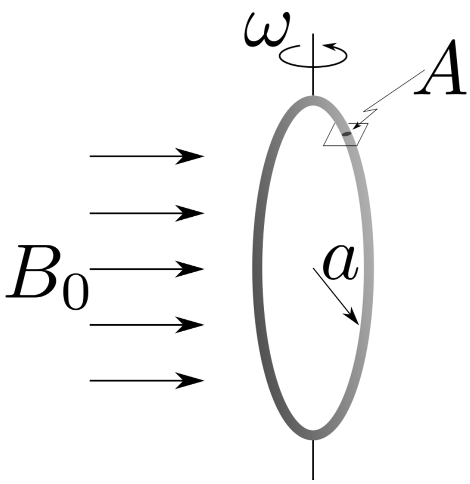
Problem 60. *1994-Fall-EM-U-3* ID:EM-U-179

A thin metal ring of radius a and cross sectional area A rotates about an axis perpendicular to a uniform magnetic field B_0. Its initial frequency of rotation is ω_0. Let the conductivity of the metal be σ, and its mass density be ρ. Recall the moment of inertia for a ring of mass m and radius r is $I = \frac{1}{2}mr^2$.

1. Find the current induced in the ring at angular velocity ω.

2. Obtain an expression for the time taken for the rotational frequency to decrease to $1/e$ of its original value, under the assumption that the energy goes into Joule heat.

   HINT: $\langle \sin^2 \theta \rangle = 1/2$, assume decay time $\gg$ rotational period.



Problem 61. *1994-Spring-EM-U-1* ID:EM-U-182

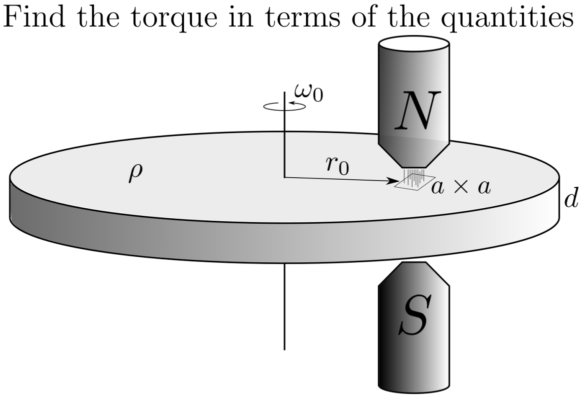
Two concentric spheres have radii a and $b > a$. The sphere with radius a has a total charge Q uniformly distributed over its surface. The sphere of radius B has charge $-Q$ uniformly distributed over its surface.

1. Find the potential everywhere
   (a) $r < a$,
   (b) $a \leq r \leq b$,
   (c) $b < r$.
2. Find the total electrostatic energy of this configuration.

Problem 62. *1994-Spring-EM-U-2* ID:EM-U-185

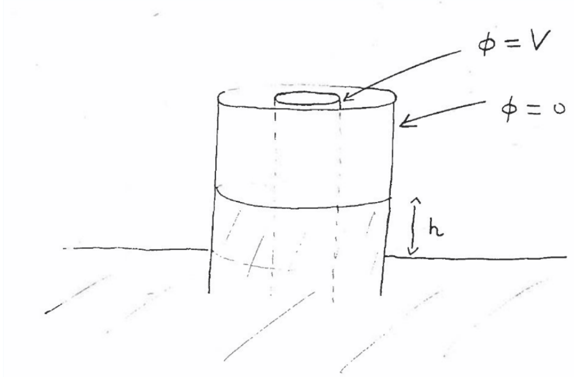
Calculate the torque tending to slow the rotation of the conducting disk shown below. The braking is generated by turning on a small region of uniform magnetic field B_0 over an area $a \times a$ at a distance of r_0 from the axis of rotation of the disk. The disk is initially rotating at an angular velocity of ω_0, has a thickness d and a resistivity ρ.

Find the torque in terms of the quantities given.



Problem 63. *1994-Spring-EM-U-3* ID:EM-U-188

A cylindrical capacitor, consisting of coaxial conducting cylinder of radii a and $b > a$, is lowered vertically into a liquid of dielectric constant ε and mass density ρ. If the liquid rises up the capacitor by a height h when a potential difference V is applied, what is the value of the dielectric constant ε?



Problem 64. *1995-Fall-EM-U-1* ID:EM-U-191

A capacitor is made from two concentric metal cylinders. The outer cylinder has an inner radius R_2 and the inner cylinder has an outer radius R_1. The gap between the cylinders is filled with a dielectric whose dielectric strength (breakdown field) is E_0.

1. Given R_2, what choice of R_1 will allow a maximum potential difference between conductors before breakdown of the dielectric? Write down an expression for the maximum potential difference V_0 in terms of R_2 and E_0.

2. Given R_2, what choice of R_1 will allow a maximum energy per unit length (of the cylinders) to be stored in the capacitor before breakdown of the dielectric? Write down an expression for the maximum energy stored per unit length U_0 in terms of R_2 and E_0.

3. Take $R_2 = 1.0$cm and evaluate R_1, V_0 for case first case, and R_1 and U_0 for the second case when the dielectric is air for which $E_0 = 3 \times 10^6$N/C. Be certain to properly specify the units of your answers.

   Given: $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \mathrm{Nm}^2/\mathrm{C}^2$.

Problem 65. *1995-Fall-EM-U-2.jpg* ID:EM-U-194

A metal ladder of length L and mass M is leaning against a building as shown in the diagram. The building and the ground are assumed to be conducting with a circuit resistance R. The ladder maintains electrical contact with both the wall and the ground as it falls. A magnetic field B is directed into the plane of the diagram.

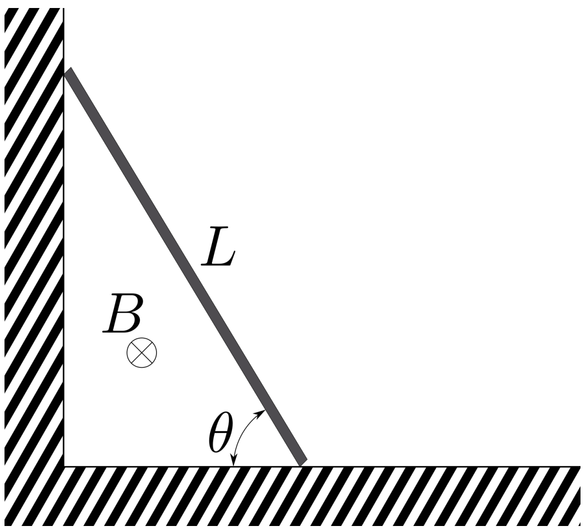
1. Aside from questions of magnitudes, will the magnetic field help stabilize the ladder in <u>static</u> equilibrium before it starts falling. Why?

2. What will be the current induced in the ladder when the angle θ at the base of the ladder is decreasing at a rate $-\omega$? Describe the physical direction of the current as the ladder falls.

3. What is the electrical power dissipated in the resistance as the ladder is falling in terms of the given quantities and ω?

4. Find the force on the ladder produced by the magnetic field as a function of ω and the given quantities. State the physical direction of this force for various values θ .

5. Describe qualitatively the effect of the magnetic field upon the falling motion if it starts from an angle near ninety degrees. In particular, does it aid or react against the falling motion as θ varies?



Problem 66. *1995-Fall-EM-U-3* ID:EM-U-197

The pre-Maxwell Equations in integral form are:

$$\oint \vec{D} \cdot d\vec{S} = Q$$

$$\oint \vec{B} \cdot d\vec{S} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\dot{\Phi}$$

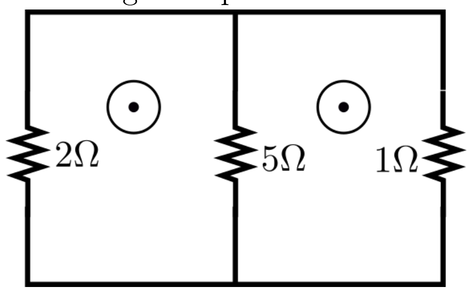
$$\oint \vec{H} \cdot d\vec{l} = I$$

1. Derive the differential form of these equations using standard vector calculus (HINT: recall $Q = \int \rho dV$, $\Phi = \vec{B} \cdot d\vec{S}$, $I = \int \vec{J} \cdot d\vec{S}$).

2. How are these equations inconsistent and what can you do to make them, consistent (Maxwell's contribution!)?

Problem 67. *1995-Spring-EM-U-1* ID:EM-U-200

In the circuit below the magnetic flux (out of the paper) changes with time t according to the equation $\phi = a + bt$, where in the left loop $a = 152$Wb and $b = 9$Wb/s and in the right loop $a = -47$Wb and $b = 2$Wb/s. Find the current in each resistor.



Problem 68. *1995-Spring-EM-U-2* ID:EM-U-203

1. The earth can be regarded as a conducting sphere of radius 6.4×10^6m; it also carries a net electric charge of $+4.5 \times 10^5$C. What is the electric field E_0 at the surface of the earth?

   The earth's atmosphere is also an electrical conductor because it contains free charge carriers that are produced throughout the atmosphere by cosmic ray ionization. The resulting free charge density ρ_0 is a constant in both space and time. The resulting conductivity $\sigma(z)$ and electric field $E(z)$ are independent of horizontal position; they only depend on the vertical coordinate z. For simplicity, for the rest of this problem, assume the surface of the earth is perfectly flat, but that E_0, is still given as in the previous part.

2. Use the fact that ρ_0 is constant together with the appropriate Maxwell equation to determine $E(z)$.

3. Write down an expression for the divergence of the current density $\vec{j}$ in the atmosphere. (Hint: Recall the continuity equation.)

4. Write down the relationship between $\vec{j}$ and σ. Use this together with the result of the previous part to find a differential equation for $\sigma(z)$.

5. Use these results to determine $\sigma(z)$. Assume $\sigma(0) = C_0$.

Do not concern yourself with the mechanism which maintains the earth's charge.

Problem 69. *1995-Spring-EM-U-3* ID:EM-U-206

A thin disk of radius a and thickness d is magnetized uniformly along its axis. Derive an expression for the magnetization M in terms of the magnetic induction B which is measured at point P. The point P is at a distance h above the center of the disk.

Problem 70. *1996-Fall-EM-U-1* ID:EM-U-209

A point magnetic dipole, mass m, dipole moment $\vec{\mu}$, is placed at a distance x above a semi-infinite superconducting slab. A superconductor has the property that it excludes all magnetic field from its interior. The dipole is oriented so that it is perpendicular to the surface of the slab. Calculate the height x at which the dipole is levitated. (Assume the experiment is carried out on the surface of the earth, and the slab is horizontal) Is the perpendicular orientation stable or unstable? Explain.

Problem 71. *1996-Fall-EM-U-2* ID:EM-U-212

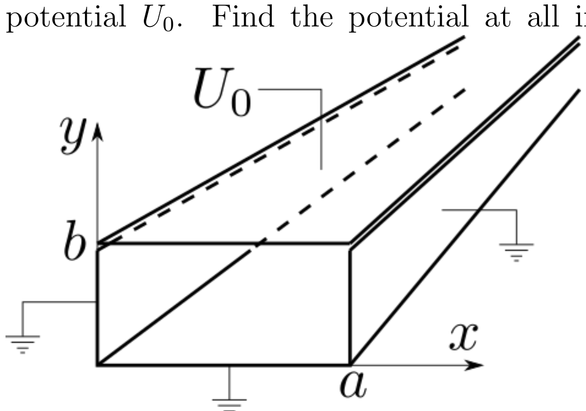
An electromagnetic plane wave is traveling in vacuum in the $\hat{x}$ direction, polarized with its E field in the $\hat{y}$ direction. It has a frequency 1GHz and an intensity $10^3\mathrm{W/m^2}$. It arrives at a semi—infinite slab of dielectric, having dielectric constant ε. The slab is oriented at 45°, with its normal vector in the direction $\vec{n} = (\hat{x} + \hat{y})/\sqrt{2}$. Calculate the intensity, polarization, and wave vector of transmitted and reflected waves.

Problem 72. *1996-Fall-EM-U-3* ID:EM-U-215

The spin of an electron is $\hbar/2$. Assuming that this is the angular momentum, find the semiclassical magnetic dipole moment of the electron in units of Bohr magnetons $e\hbar/2m$. Assume mat the charge, e, and the mass m are uniformly distributed over the volume of the electron, but make no assumption about the shape of the electron.

Problem 73. *1996-Spring-EM-U-1* ID:EM-U-218

Consider an infinitely long rectangular cylinder that has three sides grounded, while the fourth (imagined to be insulated from the others by very tiny gaps) is held at potential U_0. Find the potential at all interior points, $0 \le x \le a$, $0 \le y \le b$.

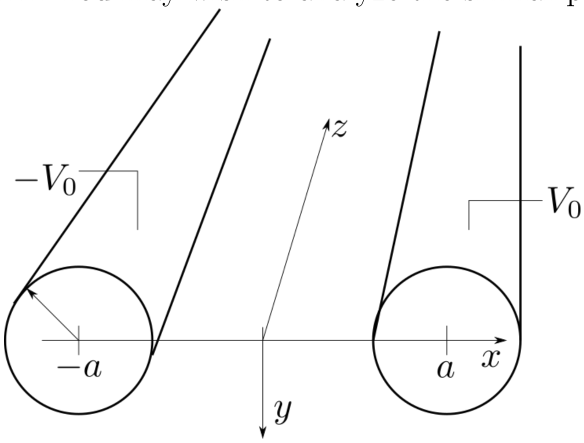


Problem 74. *1996-Spring-EM-U-2* ID:EM-U-221

Two infinite, uniform conducting cylinders of radius R are oriented with their axes parallel to the $\hat{z}$ axis. They are centered on the two points $(x, y) = (\pm a, 0, 0)$, with $a > R$. The potentials on the two cylinders are $\pm V_0$, as indicated on the figure. Find:

1. The electrostatic potential $\Phi(x, y)$ at all points in the (x, y) plane outside the two cylinders.
2. The charge per unit length on each cylinder.
3. The capacitance per unit length of the pair of cylinders.

HINT: You may wish to analyze the similar problem of two charged wires.



Problem 75. *1996-Spring-EM-U-3* ID:EM-U-224

A lightning bolt strikes the ground a short distance d from the end of and in line with a barbed wire fence. The fence has a length l and two wires separated by a distance a (vertically). Assume the two wires are connected at the ends (rain would do this on a real fence).

Assume the lightning bolt originates at a height h above the ground and can be approximated by a transient current $\vec{i}(t)$ along the vertical line segment from the cloud to the ground, with

$$\vec{i} = i_0 e^{-\left(\frac{t-t_0}{\tau}\right)^2} \hat{z}.$$

Neglect effects from the end currents in the cloud and in the ground.

1. Calculate EMF induced in the fence as a function of time.

2. Calculate the peak EMF induced and provide a numerical estimate for the following conditions:

$$\begin{aligned}
&h = 15000\text{m}\\
&d = 1\text{m}\\
&l = 30\text{m}\\
&a = 2\text{m}\\
&i_0 = 10,000\text{amps}\\
&\tau = 1\mu\text{s}\\
&\text{Use } ln(3) = 1.1, \quad ln(10) = 2.3.
\end{aligned}$$

Neglect effects from finite speed of light.

Problem 76. *1997-Fall-EM-U-1* ID:EM-U-227

In the ground state of the hydrogen atom, the electron charge is distributed with a density given by the following:

$$\rho(r) = \frac{e}{\pi a^3} e^{-2r/a}$$

where $a = 5.29 \times 10^{-11}$m is the Bohr radius and e is the charge of the electron.

1. Find the electric potential, Φ_e, and the electric field, $\vec{E}$ everywhere due to the electron charge distribution.

2. Assuming that the nucleus is a point charge of $+e$ located at the origin, find the potential, Φ_{all}, and the electric field, $\vec{E}_{\text{all}}$ due to the electron-nucleus combination at all points in space.

3. Calculate the interaction energy of the electron-nucleus system.

4. Suppose that the atom is placed in a *weak, uniform* electric field, E_w. Determine the displacement of the proton from the center of the electron cloud.

   NOTE weak means that $z \ll a$ and that you may also assume that the electron cloud is not distorted due to this field.

   In MKS units the polarizability α is given by $p = \alpha\varepsilon_0 E$, where p is the induced dipole moment and ε_0 is the permittivity of vacuum. Using the assumptions from the previous part, calculate the polarizability of the hydrogen atom in a weak electric field.

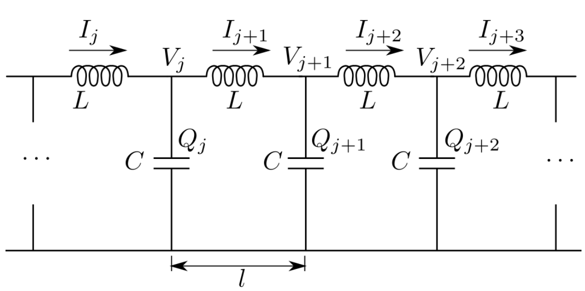
Problem 77. *1997-Fall-EM-U-2* ID:EM-U-230

Consider an infinitely long transmission line which consists of inductors and capacitors hooked together as shown in the figure.

1. Find the relation between the 2nd time derivative of the voltage on the nth node point and the voltages on the $n-1$th, nth, and $n+1$th node points.

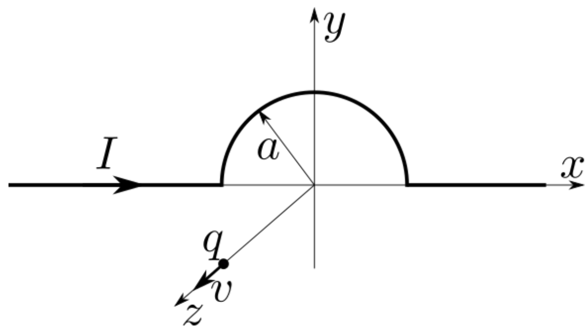
2. Find the *dispersion* relation (ω vs λ) for periodic waves of the form $V_n(t) = V_0 e^{-i(nlk-\omega t)}$ traveling down this transmission line.

3. Calculate the cutoff frequency for this system.



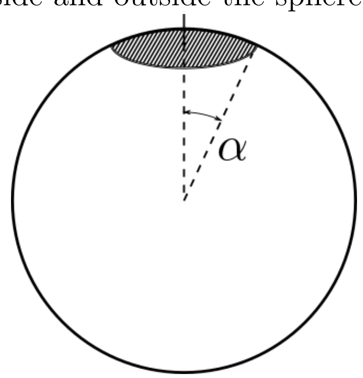
Problem 78. *1997-Fall-EM-U-3* ID:EM-U-233

A long wire carries current I along the x-axis from $-\infty$ to $-a$, then bends into a semicircle in the upper half of the $x-y$ plane, and then goes along the x-axis from a to ∞. Find the z-component of the force on a charge q at $(0,0,z)$, moving with velocity $(0,0,v)$.



Problem 79. *1997-Spring-EM-U-1* ID:EM-U-236

Consider a spherical shell of radius, R, that is electrically charged with a surface charge density, σ_0, everywhere except for $\theta \leq \alpha$ near the pole. Determine the potential inside and outside the sphere.



Problem 80. *1997-Spring-EM-U-2* ID:EM-U-239

A particle of mass, m and charge, e, moves in a spatially uniform crossed field, with perpendicular $\vec{E}$ and $\vec{B}$ fields given by:

$$\vec{E} = (E, 0, 0), \qquad \vec{B} = (0, B, 0)$$

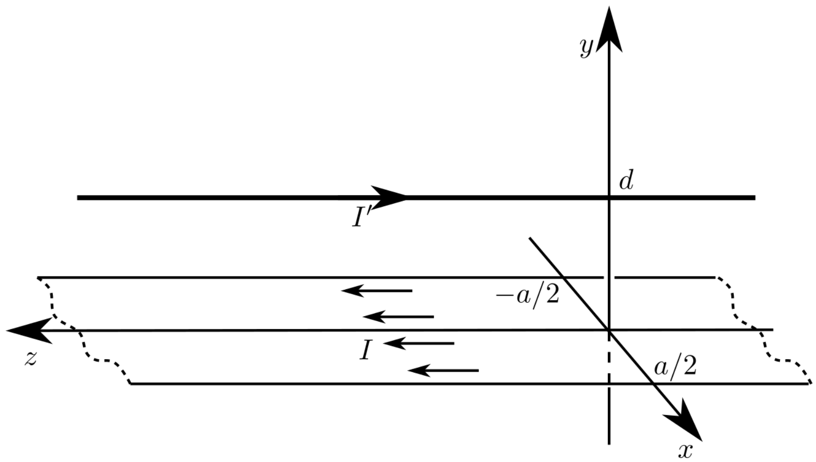
1. Write down the equations of motion for this particle.
2. Perform a Galilean (i.e. non-relativistic) transformation to an inertial frame with coordinates $(\tilde{x}, \tilde{y}, \tilde{z})$ that is moving with a velocity $\vec{v} = \frac{\vec{E}\times\vec{B}}{B^2}$ relative to the original one, and show that in this frame the equations of motion are independent of E.
3. Describe how this could be used as a velocity filter for charged particles in a directed beam, independent of their mass and charge?

Problem 81. *1997-Spring-EM-U-3* ID:EM-U-242

A very thin sheet of conducting material of infinite length and width, a, carries a uniform current, I , directed along its length. This conducting strip lies in the xz-plane and is centered on and parallel to the z-axis. A second long straight wire carrying a current of I' in the negative $z-$direction is centered on the strip and located a distance of d above the xz-plane.

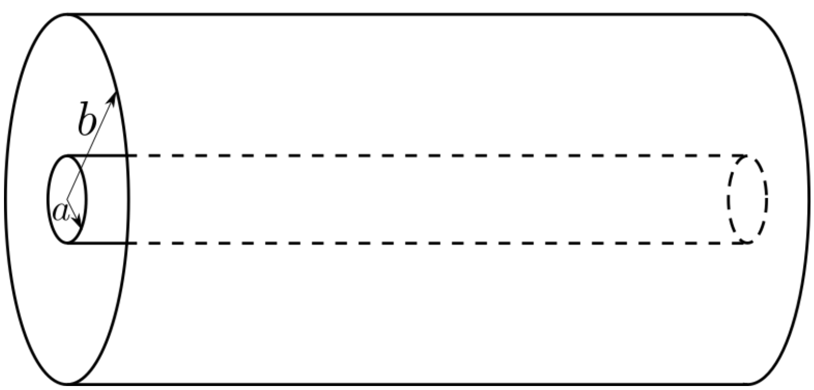
1. Find the magnetic field at any point on the y-axis.

2. Find the force on a one meter length of the thin wire due to the magnetic field of the conducting strip.



Problem 82. *1998-Fall-EM-U-1* ID:EM-U-245

A long coaxial cable consists of a conducting core of radius a and an infinitesimally thin walled tube of radius b. Current flowing in this cable is uniformly spread over the area of the inner cylinder. The cross section of the cable is shown in the figure. The space between the core and the sheath is filled with vacuum. Find the self-inductance per unit length of this cable.



Problem 83. *1998-Fall-EM-U-2* ID:EM-U-248

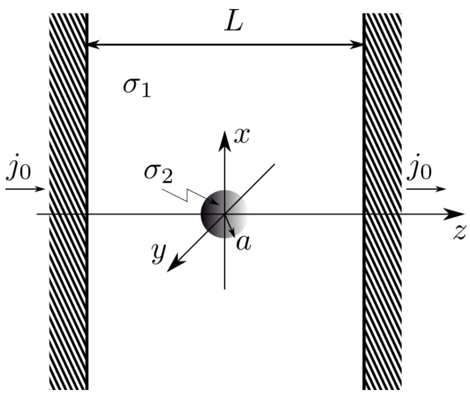
1. An infinite insulating plane of thickness d is uniformly charged with a volume charge density ρ. Find the electric potential, ϕ, everywhere. Take the z-axis to be perpendicular to the plane with $z = 0$ at the midpoint of the thickness d. Also take the potential at the origin to be your reference potential with $\phi = 0$.

2. You are given a three dimensional electric charge distribution with a volume charge density which filling all space given by $\rho(x, y, z) = \rho_0 \cos(\alpha x) \cos(\beta y) \cos(\gamma z)$, where ρ_0, α, β, and γ are constants. Find the potential $\phi(x, y, z)$ due to this charge distribution. Specify which point you take as a reference point $\phi = 0$. (Hint, use separation of variables to find the solution for $\phi(x, y, z)$.)

Problem 84. *1998-Fall-EM-U-3* ID:EM-U-251

An infinite *slab* of conducting material of thickness L and conductivity σ_1 is centered about the origin and oriented parallel to the xy-plane. At the center of this slab is a defect consisting of a small sphere of radius a with a different conductivity σ_2. The current density input to this slab in the *steady state* is uniform across the face of the slab and directed in the $+z$ direction. Assume that $a \ll L$.

1. In the *steady state* what is the relationship between the current density and the electrostatic potential?
2. Give all boundary conditions at the surface of the spherical defect that are necessary to find the current density inside the defect.
3. Calculate the current density inside the small spherical defect.



Problem 85. *1998-Spring-EM-U-1* ID:EM-U-254

A thick spherical shell with an inner radius a and outer radius b, is made of a dielectric material that has a fixed polarization given by

$$\vec{P} = \frac{k}{r}\hat{r},$$

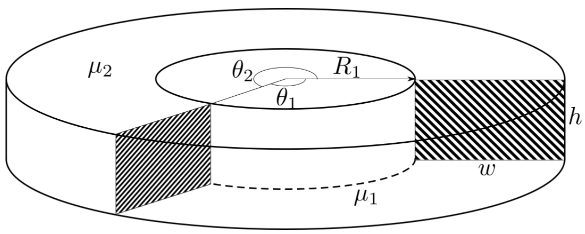
where k is a constant, r is the distance from the center of the sphere and $\hat{r}$ is the unit vector in the radial direction.

1. Determine the magnitude and location of all the bound charges for this system.
2. What is the total bound charge for this system.
3. Determine the electric displacement vector, $\vec{D}$; for the three regions:
   (a) Region 1: $0 < r < a$.
   (b) Region 2: $a < r < b$.
   (c) Region 3: $b < r$
4. Determine the potential at the center of the sphere relative to the potential at infinity.

Problem 86. *1998-Spring-EM-U-2.jpg* ID:EM-U-257

Consider an ideal toroidal magnet with a rectangular cross section A of height h and width w, is made using two different materials with permeabilities μ_1 and μ_2 respectively wrapped uniformly with total of N turns of wire each carrying a current I. The inner radius of the toroid is R_1. The material of permeability μ_1 subtends an angle of θ_1, and the material of permeability μ_2 subtends an angle of θ_2 about the center of the toroid as sketched in the figure.

1. Give the boundary conditions on $\vec{B}$ and $\vec{H}$ at the interface between the two materials.
2. Find $\vec{B}$ and $\vec{H}$ in the toroid.
3. Find the inductance L of this system.

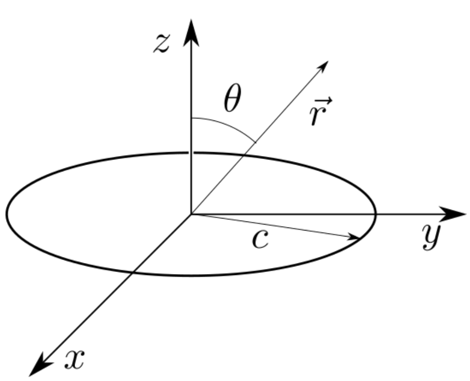


Problem 87. *1998-Spring-EM-U-3* ID:EM-U-260

A total charge Q is uniformly distributed around a ring of radius c that lies in the xy-plane, centered on the origin as shown.

1. Calculate the electrostatic potential at an arbitrary point on the z-axis.
2. Use the result from the previous part to write down the expression for the potential at an arbitrary point (r, θ) in each of the regions $r \leq c$ and $r \geq c$, as an infinite series of Legendre polynomials $P_1(\cos\theta)$.

   NOTE: An axially symmetric solution of Laplace's Equation can be expanded in the form:
$$\phi(r,\theta) = \sum_{l\geq 0} \left(a_l r^l + b_l r^{-l-1}\right) P_l(\cos\theta).$$



Problem 88. *1999-Fall-EM-U-1* ID:EM-U-263

A long straight wire of radius R is coaxial with the z axis, with a current density given by $\vec{J} = (0, 0, a/\rho)$, for $\rho \leq R$, where a is a constant and $\rho = \sqrt{x^2 + y^2}$. Find a vector potential $\vec{A}$ and the magnetic field $\vec{B}$, both inside and outside the wire.

Problem 89. *1999-Fall-EM-U-2* ID:EM-U-266

Consider two infinite line charges, each with charge per unit length λ, but of opposite signs, that run parallel to the z axis at $x = l$ and $x = -l$.

1. First, find the electric field due to just the wire with positive charge density, at an arbitrary distance from the wire.
2. Find the potential (relative to the origin) due to both wires, at a distance r from the origin in the xy-plane.
3. Find the leading-order r-dependence of the potential for large values of r ($r \gg l$).

Problem 90. *1999-Fall-EM-U-3* ID:EM-U-269

A parallel-plate capacitor consists of two circular plates of radius a, separated by a small gap d, where $d \ll a$. Charges Q_0 and $-Q_0$ are placed on the two plates, and, at time $t = 0$, their centers are connected with a thin straight wire of resistance R. Assume that R is very large, so that at any time the field across the plates remains uniform, and inductance can be neglected.

1. Calculate the charge on each plate as a function of time.
2. Calculate the total current crossing a ring of radius ρ (where $\rho < a$) in either of the plates, as a function of time. (The ring is concentric with the centre of the plate.)
3. Calculate the magnetic field between the plates, as a function of time and of the radial distance from the center. (Hint: Construct an argument, based on symmetry principles, for why the magnetic field will be purely azimuthal. To get full marks, you must *explain* why this is true.)

Problem 91. *1999-Spring-EM-U-1* ID:EM-U-272

A capacitor consists of two square parallel conductors of side a separated by a distance d. A piece of uniform dielectric material, with dielectric constant ϵ, precisely fills the volume between the plates, and is free to slide without friction. A potential difference V between the plates is established by temporarily connecting a battery, and then disconnecting it.

1. Calculate the force on the dielectric if it is displaced by a distance $x < a$ (keeping its edges parallel to the edges of the conductors). Calculate also the leading-order expression for the force when x is very small compared with a.

2. Repeat the calculations of the previous part for the case where the battery is instead left connected to the capacitor, causing the potential difference V to be held fixed.

3. Compare your answers from parts the previous parts, both for small displacements ($x \ll a$) and for arbitrary displacements with $x < a$.

Note: Ignore edge effects in all calculations.

Problem 92. *1999-Spring-EM-U-2* ID:EM-U-275

A long cylindrical conductor of radius R has a cylindrical hole of radius a bored down its length, with its axis displaced by a distance b from the axis of the conductor. A current I flows down the conductor, distributed uniformly across the conductor.

Obtain an expression for the magnetic field $\vec{B}$ inside the hole, and show that it is uniform.

Problem 93. *1999-Spring-EM-U-3* ID:EM-U-278

1. Show that the vector potential due to a static current density $\vec{J}$ can be written as

$$\vec{A}(\vec{r}) = \frac{1}{c}\int \frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|} d^3\vec{r}'$$

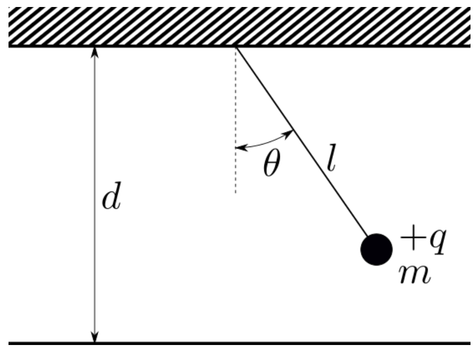
   (Hint: You can prove this by starting from the Biot-Savart Law for the magnetic field due to a current element. Alternatively, you can derive a differential equation satisfied by $\vec{A}$, and write down its solution by making an analogy with the solution for the electrostatic potential in terms of the charge density, whose solution you may assume.)

2. Consider an infinite, thin conducting wire, lying along the entire z axis, carrying a current I . Write down the current density $\vec{J}(x, y, z)$ that describes this situation, and use the result from the previous part to obtain an integral expression for the vector potential. Show that this expression diverges.

3. Using the expression for $\vec{A}$ obtained in the previous part, show that $\vec{\nabla} \times \vec{A}$ is a convergent integral, and by evaluating, it show that it correctly describes the magnetic field outside an infinite wire.

Problem 94. *2000-Fall-EM-U-1* ID:EM-U-281

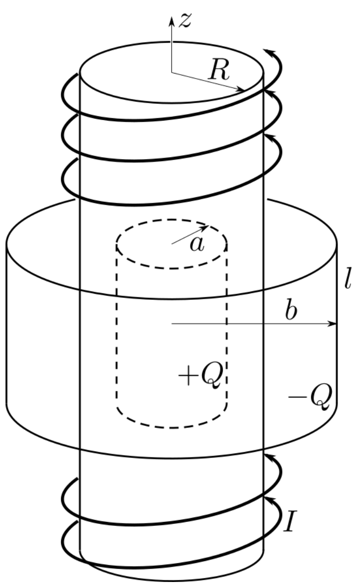
A particle with mass m and charge $+q$ is suspended by a string of length l. At a distance d below the point of suspension there is an infinite plane conductor. Determine the frequency of the pendulum for small oscillations about its equilibrium point. (Neglect gravity and radiation damping and the upper surface.)



Problem 95. *2000-Fall-EM-U-2* ID:EM-U-284

A very long solenoid, carrying current I, has radius R and N turns per unit length. Located inside the solenoid, and concentric with it, is a conducting cylindrical shell of radius a ($a < R$) and length l. Also concentric with the shell and the solenoid is another conducting cylinder of radius b ($b > R$) and length l. A charge $+Q$ is uniformly distributed on the inner cylinder and a charge of $-Q$ is on the outer one. Both cylinders are free to rotate on their common axis.

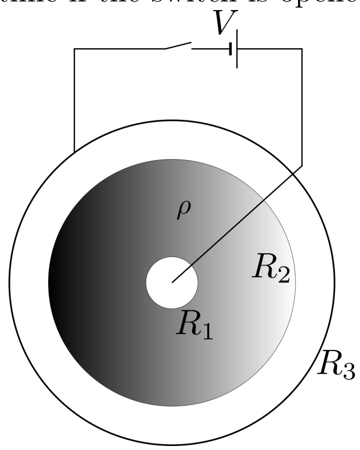
1. What fields exist between the cylinders at a steady state?
2. What angular momentum is stored in the fields?
3. If the current in the solenoid is turned off, what is the induced electric field as a function of dI/dt?
4. What angular momentum gets imparted to each cylinder after the current is turned off?



Problem 96. *2000-Fall-EM-U-3* ID:EM-U-287

Consider a large sphere of radius R_3. From its center out to radius R_1 , it is made of material with zero resistivity. From R_1 to R_2 it is made of material with resistivity ρ, and from R_2 to R_3 the material again has zero resistivity. A tiny hole is drilled into the sphere and a battery with voltage V is connected as shown. Assume that this tiny hole does not affect the symmetry and that the current flows out uniformly.

Ignoring self-inductance, find the current flowing through the sphere as a function of time if the switch is opened at $t = 0$.



Problem 97. *2000-Spring-EM-U-1* ID:EM-U-290

Consider a parallel—plate capacitor that is part of a simple $R - C$ circuit connected to a battery of e.m.f. $\mathcal{E}$ and resistance R. The battery is then disconnected, and a slab of material of thickness b is placed between the capacitor plates. The plates have area A and the separation between them is d ($d > b$).

At room temperature, the slab is a good dielectric, with dielectric constant κ. As the temperature increases beyond a critical point, the slab becomes a good conductor.

Assume that the transition takes place uniformly, in a. very short time τ.

1. What is the capacitance of the system at room temperature with the added material?

2. How much external work is involved in the process of inserting the slab?

Next, the battery is reconnected to the capacitor:

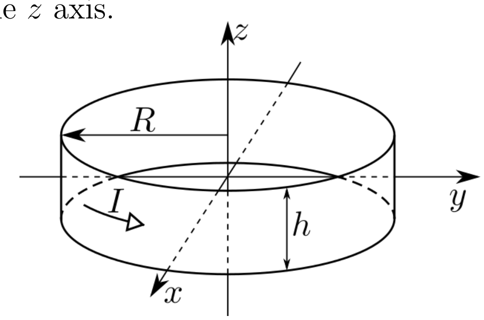
3. How much power is dissipated in the resistor after the circuit is restored?

4. If the operating temperature exceeds the critical temperature for a transition to the conducting state, what is the capacitance of the system?

5. Find the power dissipated in the resistor when the dielectric makes the transition to its conducting state.

Problem 98. *2000-Spring-EM-U-2* ID:EM-U-293

You are given a conducting band which carries an azimuthal current I , uniformly spread through its cross section. The band has a height h, and a radius R. It is infinitesimally thin. The band is centered on the z axis, and the (x, y) plane divides the band in half. Find the magnetic field due to the current band at any point on the z axis.



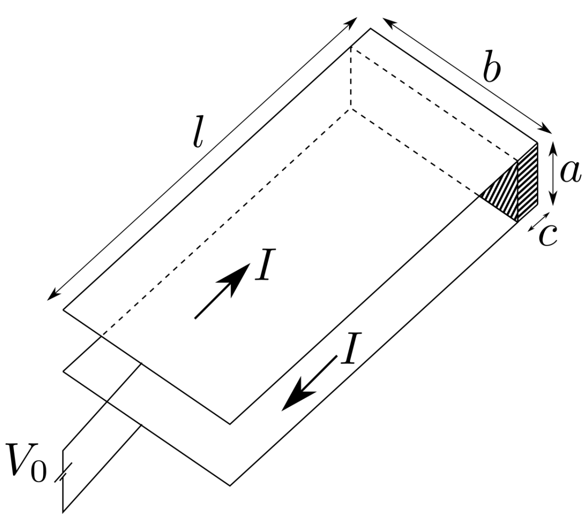
Problem 99. *2000-Spring-EM-U-3* ID:EM-U-296

The system shown in the figure consists of two flat conducting strips of length l, width b, separated by a small gap a ($a \ll b \ll l$). The right ends of the strips are connected by a resistive material (resistivity ρ), and a battery of voltage V_0 is connected across the left ends. The current is assumed to be uniform across the strip, and to flow only parallel to the l-dimension of the strips. Neglect all resistances of the strips and wires, and all effects arising from the finite speed of propagation of electromagnetic fields.

1. Find $\vec{E}$ and $\vec{B}$ everywhere between the strips. (Neglect edge effects, and look only in the region outside the resistive material.)

2. Find the Poynting vector $\vec{S}$.

3. In which direction does the electromagnetic energy flow, and where does it go?



Problem 100. *2001-Fall-EM-U-1* ID:EM-U-299

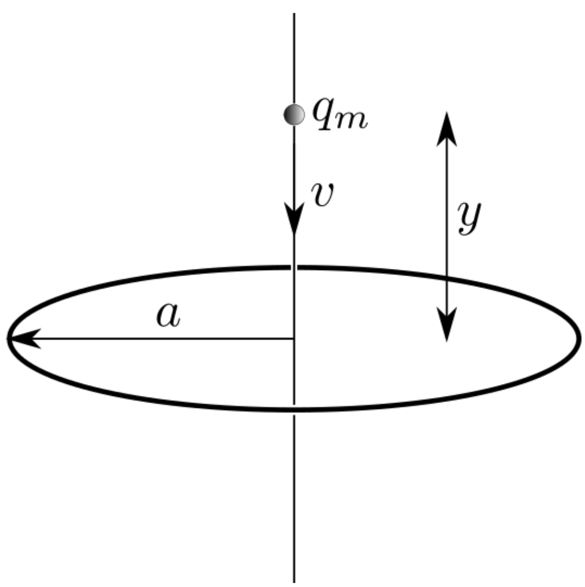
Two thin, parallel, infinitely long, non-conducting rods are a distance a apart, They each carry a constant charge density per unit length λ in their rest frame. Calculate the force per unit length between them:

1. in their rest frame;
2. in a lab frame in which the rods are moving along themselves with velocity v, not necessarily small compared to the speed of light. Compare the results.

Problem 101. *2001-Fall-EM-U-2* ID:EM-U-302

A magnetic monopole of strength q_m moves downward along the axis of a conducting ring with radius a. [For a magnetic monopole $|\vec{B}| = k_m q_m / r^2$, where $k_m = \mu_0/4\pi$.]

1. Find the magnitude of the magnetic flux through the ring when the monopole is a distance y above the ring.
2. Find the induced emf from the previous part if the velocity of the monopole at y is v. Indicate the direction in which an induced current will circulate.
3. In mks units, the Dirac quantization condition for the minimum: q_m is $ek_m q_m = h/4\pi$. Evaluate q_m (in A·m). Evaluate the emf (in V) for $a = 1$cm, $y = 0$ and $dy/dt = c/10$, where c is the speed of light.



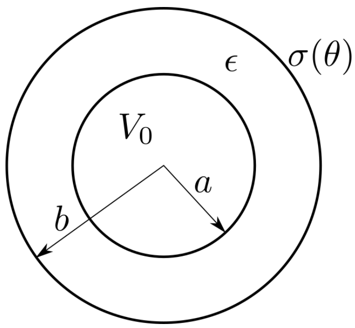
Problem 102. *2001-Fall-EM-U-3* ID:EM-U-305

A sphere, centered at the origin, with radius R and uniform volume charge density ρ is spinning with angular frequency ω about the z-axis. Find the resulting magnetic field $\vec{B}$ along the z-axis for $z > R$.

Problem 103. *2001-Spring-EM-U-1* ID:EM-U-308

A conducting sphere of radius a, at a potential V_0, is surrounded by a thin concentric spherical shell of radius b, over which there is a surface charge $\sigma(\theta) = k\cos(\theta)$. Here k is a constant and r, θ, and ϕ are the usual spherical coordinates. The region between the outer shell and the inner sphere is filled with a linear dielectric material with permittivity ϵ.

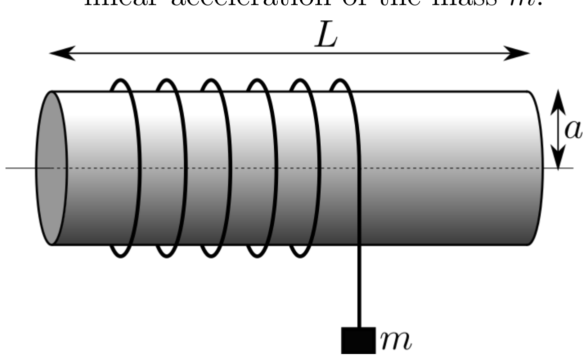
1. Find the potential for $r > b$ and for $a < r < b$.
2. Find the total surface charge (bound plus induced), $\sigma_{\text{sphere}}(\theta)$, on the conducting sphere.
3. What is the total charge of this system?



Problem 104. *2001-Spring-EM-U-2* ID:EM-U-311

A cylinder of moment of inertia I, radius a and length $L \gg a$ has a fixed uniform surface charge density, σ. It can rotate about the x-axis. A mass m hangs from a rope wrapped around the cylinder (assume no slippage).

1. a) If the cylinder rotates at angular velocity $\omega = d\theta/dt$, find the surface current density, K (K is defined per unit length along the x-axis.)
2. For a given K, find the field B_x along the x-axis.
3. If the rotation rate is not constant, find the tangential component of the induced electric field.
4. Find the torque on the cylinder due to this electric field.
5. From the equations of motion for the mass m and for the cylinder, find the linear acceleration of the mass m.



Problem 105. *2001-Spring-EM-U-3* ID:EM-U-314

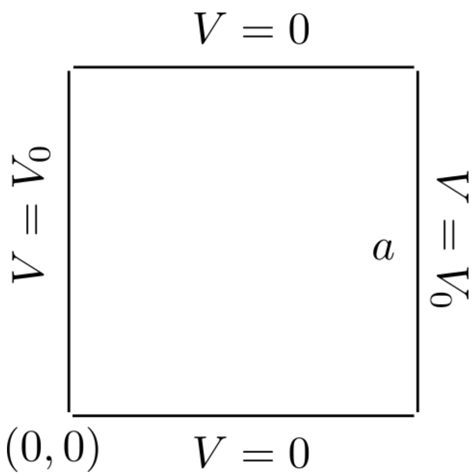
A finite—length solenoid of radius R, with n windings per meter and length L, is centered on the origin with its long axis along the z-axis. It carries current I.

1. Derive the expression for the magnetic field at any point on the z-axis inside the solenoid.
2. Show that for $R \ll L$ this becomes the standard position—independent result for an infinite solenoid.

2 Graduate level

Problem 106. *1983-Fall-EM-G-4* ID:EM-G-2

1. Draw the lines of force and equipotentials inside this square of side a in two dimensions. Each side of the square is held at a fixed potentiaI, as shown.
2. Solve for $V(x, y)$ within the square. Then find the charge density on the lower edge.



Problem 107. *1983-Fall-EM-G-5* ID:EM-G-5

A distribution of charge has density

$$\rho(r, \theta, \phi) = \frac{1}{64\pi} r^2 e^{-r} \sin^2(\theta).$$

1. Write the charge distribution as an explicit expansion in $P_l(\cos(\theta))$, determining the coefficient of each P_l.
2. Which of the following multipole moments of the electric potential will be non-zero:
   (a) monopole,
   (b) dipole,
   (c) quadrupole,
   (d) octupole,
   (e) hexadecapole.
3. Determine the spherically symmetric part of the electric potential for all r, choosing the zero of potential to lie at infinite r.

You can use

$$P_0(z) = 1, \qquad P_1(z) = z, \qquad (l+1)P_{l+1}(z) = (2l+1)zP_l(z) - lP_{l-1}(z).$$

Problem 108. *1983-Fall-EM-G-6* ID:EM-G-8

A certain electromagnetic field is produced by the potentials

$$\begin{aligned}\phi &= 0\\ \vec{A} &= -(ct/r^2)\hat{r}\end{aligned}$$

1. Calculate $\vec{E}(r,t)$, $\vec{B}(r,t)$.
2. Find a $\phi_1(r,t)$ which together with $\vec{A}_1(r,t) = 0$ will give the same $\vec{E}$ and $\vec{B}$ fields. How unique is this $\phi_1(r,t)$?
3. Write down the gauge transformation relating $\phi(r,t)$, $\vec{A}(r,t)$ to $\phi_1(r,t)$, $\vec{A}_1(r,t)$.

Problem 109. *1983-Spring-EM-G-4* ID:EM-G-11

Given a medium in which $\rho = 0$, $\vec{J} = 0$, $\mu = \mu_0$ but where the polarization $\vec{P}$ is a given function of position and time: $\vec{P} = \vec{P}(x,y,z,t)$. Show that the Maxwell equations are correctly obtained from a single vector function $\vec{Z}$ (the Hertz vector), where $\vec{Z}$ satisfies the equation

$$\Delta\vec{Z} - \frac{1}{c^2}\frac{\partial^2\vec{Z}}{\partial t^2} = -\frac{\vec{P}}{\varepsilon_0},$$

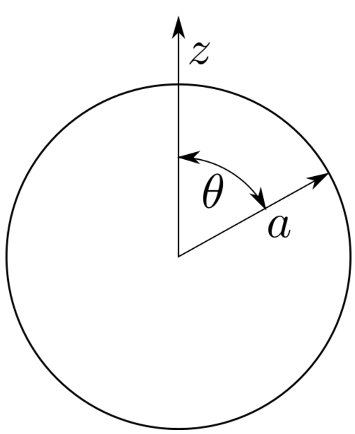
and

$$\vec{E} = \mathrm{curlcurl}\vec{Z} - \frac{1}{\varepsilon_0}\vec{P}, \qquad \vec{B} = \frac{1}{c^2}\mathrm{curl}\frac{\partial\vec{Z}}{\partial t}.$$

Problem 110. *1983-Spring-EM-G-5* ID:EM-G-14

1. Calculate the potential both inside and outside the charged spherical shell of radius a shown below.

   The potential on the sphere is $V_0+V_1\cos^2(\theta)$. (Hint: $P_0(\cos(\theta)) = 1$, $P_1(\cos(\theta)) = \cos(\theta)$, $P_2(\cos(\theta)) = \frac{1}{2}\left(3\cos^2(\theta) - 1\right)$.)
2. Calculate the charge density on the sphere.



Problem 111. *1984-Fall-EM-G-4* ID:EM-G-17

A metal rod of length L rotates about one of its ends about an axis perpendicular to the rod with an angular velocity ω. A uniform magnetic field $\vec{B}$ is parallel to this rotation axis. What is the induced emf across the ends of the rod?

Problem 112. *1984-Fall-EM-G-5* ID:EM-G-20

Space is divided into two parts by a grounded conducting plane at $z = 0$. On one side of the conducting plane is a point charge q at $(0, 0, a)$. 0n the other side there are no charges.

1. Calculate the electric potential and the electric field everywhere in space.
2. Calculate the work done by the electric field in bringing the charge q from infinity to a distance d from the conducting plane. Compare this with the potential energy the charge has at this position.

Problem 113. *1984-Spring-EM-G-4* ID:EM-G-26

A spherical shell of radius R has a charge distribution $\sigma(\theta, \phi)$, where θ and ϕ are the usual spherical angles. For $r > R$ this distribution generates,a potential given by $V(r) = \frac{k}{r^2}\cos(\theta)$.

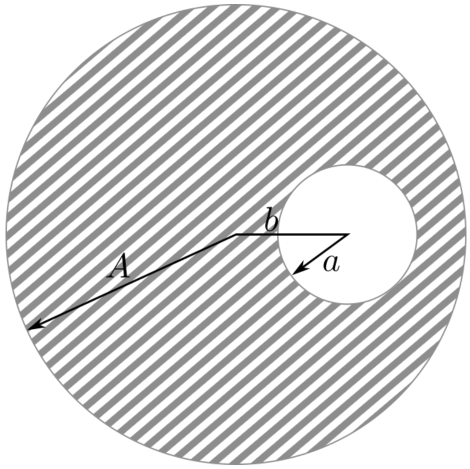
Find $V(r)$ for $r < R$ and $\sigma(\theta, \phi)$.

Problem 114. *1984-Spring-EM-G-5* ID:EM-G-29

A conductor is made by taking a solid cylinder of radius A and removing from it a cylinder of radius a. The axis of the hole is offset from the axis of the original solid cylinder by a distance $b > a$. A cross-section of the final conductor is shown on the figure. The length of the conductor is much larger than A.

A current I flows in this conductor. No current flows in the hole.

1. Calculate the magnetic field outside the conductor $r > A$.
2. Calculate the magnetic field inside the conductor but outside the hole.
3. Calculate the magnetic field inside the hole.
4. Draw a diagram which shows the field lines in each of the three regions.



Problem 115. *1985-Fall-EM-G-4* ID:EM-G-32

The development of grand unified theories has 1ed to a great deal of interest in the possible existence of magnetic monopoles.

1. Using Gauss1an (CGS) units, write down the appropriate modified Maxwell's equations when magnetic monopoles are assumed to exist. You may assume $\mu = \epsilon = 1$.

2. A simple magnetic monopole detector consists of a single superconducting loop of radius R. Assume the loop is located in the $x - y$ plane and centered on the origin. A magnetic monopole, with magnetic charge g, is moving along the line $y = z$, $x = 0$ at a speed V. What is the change in the magnetic flux enclosed by the superconducting ring when the monopole passes the origin.

Problem 116. *1985-Fall-EM-G-5* ID:EM-G-35

A solid spherical conductor of radius a is coated with an insulating material with dielectric constant ε to an outer radius b. The region $r > b$ is vacuum. $\Phi = V$ on the sphere and $\phi \to 0$ at $r \to \infty$. Find:

1. The potential everywhere in space.

2. The total charge on each surface.

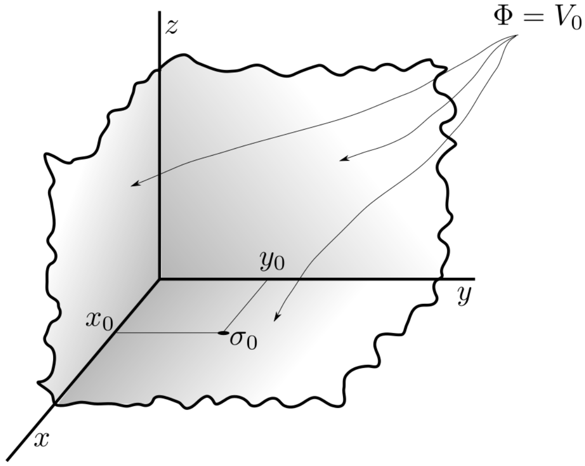
3. The capacitance of the conducting sphere. Is it larger or smaller than that of an uncoated sphere?

Problem 117. *1985-Fall-EM-G-6* ID:EM-G-38

Consider the region $x \geq 0$, $y \geq 0$, $z \geq 0$ (i.e., the region bounded by the $x - y$, $x - z$, and $y - z$ planes).

1. Find the most general solution to Laplace's equation which may be expressed in the form $\Phi = A(x)B(y)C(z)$ when the separation constants are all equal to zero.

2. Assume $\Phi = V_0$ on each of the three planes bounding the region $x, y, z, \geq 0$ and the charge density on the $x-y$ plane at the point $(x_0, y_0, 0)$ is σ_0. Use your answer to the previous part to find a solution for $\Phi(x, y, z)$ for $x, y, z \geq 0$.



Problem 118. *1985-Spring-EM-G-4* ID:EM-G-41

A dipole $\vec{p}$ is a distance d from an infinite, conducting plane. The dipole is oriented at an angle θ with respect to the normal to the plane.

What are the force and torque exerted on the dipole by the plane?

Problem 119. *1985-Spring-EM-G-5* ID:EM-G-44

Show that the normal derivative of the electric field at the surface of a conductor satisfies:

$$\frac{1}{E}\frac{\partial E}{\partial n} = -\left(\frac{1}{R_1} + \frac{1}{R_2}\right),$$

where R_1 and R_2 are the radii of curvature of the surface.

Problem 120. *1985-Spring-EM-G-6* ID:EM-G-46

1. We would like to construct a superconducting solenoid to produce a 10T magnetic field with a current of 100Amps. Give a rough estimate of the inductance of this solenoid, assuming its inner dimensions are 10cm radius by 1m long.

2. A 5 volt power supply is available. How long will it take to energize the magnet, assuming the resistance of the leads is negligible?

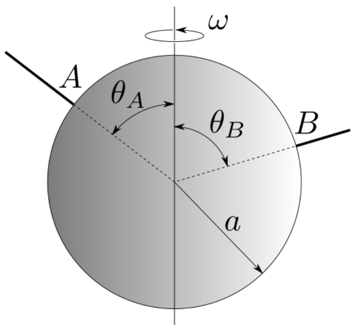
3. Approximately how much energy is stored in the magnet?

4. If the magnet has a shunt resistance $= 10^5\Omega$, by approximately how much would the current decay over a period of one hour?

Problem 121. *1986-Spring-EM-G-4* ID:EM-G-47

Consider a thin spherical shell with radius a, thickness t, and conductivity σ, centered on the origin. The shell is spinning about the z axis with angular frequency ω. There is a uniform magnetic field $\vec{B} = B\hat{z}$ throughout space. Two brushes, attached to stationary wires, lightly touch the spheres. One brush touches point A at an angle θ_A with respect to the z axis, while the other brush touches point B at angle θ_B. The points A and B are separated by π radians in the azimuthal (ϕ) direction.

1. What is the emf developed between two wires?
2. If $\theta_A < \theta_B < \pi/2$, what is the sign of $V_B - V_A$? Be sure to justify your result.



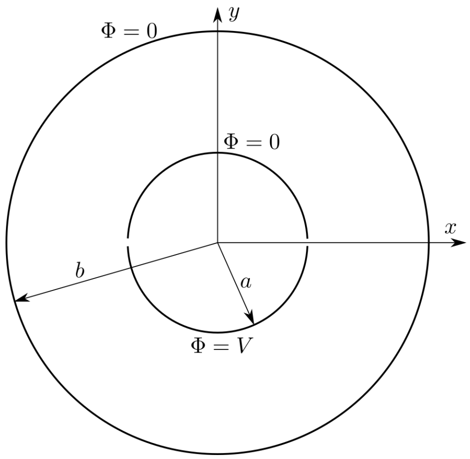
Problem 122. *1986-Spring-EM-G-5* ID:EM-G-50

1. Consider a loop of wire in the $x - y$ plane, centered on the origin, with radius r. A current I is flowing through the wire. Find the magnetic field $B(z)$ at any point along the z axis.
2. Use your result from the previous part to find the magnetic field at any point alone the axis of solenoid with inner radius a, outer radius b, length $2L$, and a current per unit area J_0. To be specific, assume, the axis of the solenoid is the z axis, and that it extends from $z = -L$ to $z = L$.

Problem 123. *1986-Spring-EM-G-6* ID:EM-G-53

Consider the potential in the region between two infinite concentric conducting cylinders with radii a and b. The outer cylinder is at potential $\Phi = 0$. The inner cylinder has a thin insulating sheet inserted in the $y = 0$ plane, see figure. Two half—cylinders are at $\Phi = 0$ and $\Phi = V$. The region between the two cylinders is vacuum.

1. Find the potential $\Phi(\vec{r})$ in the region between the two cylinders, expressed in terms: of the cylindrical coordinates of the point $\vec{r}$.
2. Find the total charge per unit length on the surface at $\rho = b$. (Hint: You do not need to determine the ϕ dependence.)



Problem 124. *1987-Fall-EM-G-4* ID:EM-G-56

1. Find the electric field at all points inside and outside a hollow perfectly conducting spherical shell of inner radial R_1. and outer radius R_2 which is placed in a uniform electric field $\vec{E} = E_0\hat{z}$.

2. Find the surface charge density on the inside and outside surface of the spherical shell.

3. Find the electric field at all points in space if a point charge q is also placed at the center of the hollow conducting shell.

Problem 125. *1987-Fall-EM-G-5* ID:EM-G-59

An infinitely long cylinder of radius R and permeability μ is placed in an external uniform magnetic field $\vec{B}_0$, which is perpendicular to the axis of the cylinder. Find $\vec{B}$ and $\vec{H}$ everywhere.

Problem 126. *1987-Fall-EM-G-6* ID:EM-G-62

A plane wave in vacuum is incident normally on the plane surface of a dielectric medium with dielectric constant ϵ and permeability, μ. From the boundary conditions on the fields find the reflection coefficient (ratio of reflected to incident intensity) in terms of the index of refraction n and permeability μ.

Problem 127. *1988-Fall-EM-G-4* ID:EM-G-65

1. Write down the differential form of Maxwell's equations in a vacuum.

2. A perfect conducting surface is located at $y = 0$ and another is located at $y = q$ as drawn below. In the remainder of this problem we are looking for a transverse magnetic wave of frequency ω that propagates in the $\hat{z}$ direction. What form should one assume such a wave would have? What boundary conditions must the wave satisfy?

3. Now assume that $\vec{H}$ lies only in the $\hat{x}$-direction and is independent of x. By satisfying the boundary conditions and Maxwell's equations solve for the general

form of $\vec{H}$. What is the lowest frequency which will propagate between these plates?

Problem 128. *1988-Fall-EM-G-5* ID:EM-G-68

A conducting sphere of radius R is charged to a potential V and spun about a diameter at an angular velocity ω.

1. Calculate the surface charge density.
2. Now calculate the surface current density.
3. Find the magnetic induction at the center of the sphere.
4. What would be the value of B for a sphere 0.1 meter in radius, charged to 10kilovolts and spinning at 10^4 revolutions per minute?
5. Calculate the magnetic dipole moment μ for the rotating sphere.
6. For the numerical values of R, V, and ω given before, what would be the value of μ?
7. What current flowing through a loop 0.1 meter in diameter would give the same magnetic dipole moment?

Problem 129. *1989-Fall-EM-G-4* ID:EM-G-71

Consider an electromagnetic wave of frequency ω, with the electric field along the x-axis, and the magnetic field along the y-axis. Assume the spatial dependence is given by $e^{i(kz-\omega t)}$, with k unknown.

1. If the wave travels through a nonconducting dielectric medium, of dielectric constant ϵ ($\mu = \mu_0$), derive the wave equations satisfied by the fields $\vec{E}$ and $\vec{B}$. Find $k(\omega)$ and the relative amplitudes of the electric and magnetic fields.
2. Repeat the above calculations if the medium is an electrical conductor of conductivity σ and dielectric constant ϵ. For the purpose of this calculation you may work in the limit in which the displacement current is negligible. Give the criteria under which this limit is a valid approximation. What happens to $|\vec{E}|/|\vec{B}|$ as $\omega \to 0$? Approximately how far will such a wave extend into Cu, for an FM radio station at 90MHz? Take $\sigma = 6.0 \times 10^5(\text{Ohm}\cdot\text{cm})^{-1}$.

Problem 130. *1989-Fall-EM-G-5* ID:EM-G-74

Consider the potential

$$\Phi_{\text{ext}} = A(x^4 + y^4) - Bx^2y^2.$$

1. To what charge density does this correspond?

2. Setting $B = 6A$, show that Φ_{ext} can be rewritten as

$$\Phi_{\text{ext}} = A\rho^4 \cos(4\phi).$$

3. An infinite cylinder, of radius a and dielectric constant ϵ, is placed along the z-axis. Find the potential Φ_d produced by the dielectric cylinder, for $\rho < a$ and for $\rho > a$.

4. Find the surface charge density on $\rho = a$.

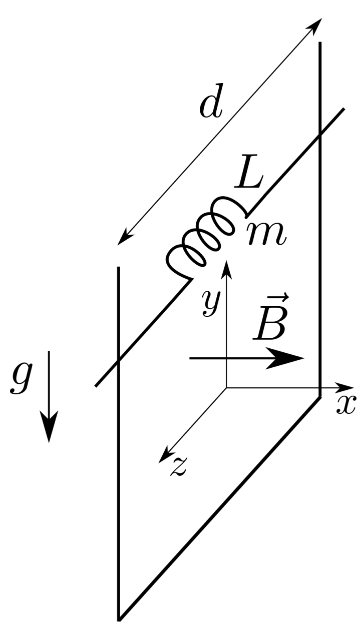
5. Find the force per unit area $\vec{f}$ on the surface charge, due to Φ_{ext}, as a function of ϕ.

Problem 131. *1989-Spring-EM-G-4* ID:EM-G-77

Consider a rod of length d, with a large coil of inductance L attached to it. Both the rod and the inductance together have mass m. The rod with inductor is in electrical contact and constrained to slide without friction along a set of vertical conducting rails. The rails are joined at the bottom of the track to form a closed loop. The force of gravity acts in the downward direction. The system is in a region of uniform magnetic field, $\vec{B}$ directed perpendicular to the plane formed by the rails. The initial velocity of the "slide" is zero, the height is y_0 and there is no current flowing in the loop. Neglect the resistance of all the conductors.

1. Set up and solve the circuit equation for I, the current flowing in the closed loop, as a function of the height of the slide y.

2. Solve the equations of motion to find $y(t)$ for all t.



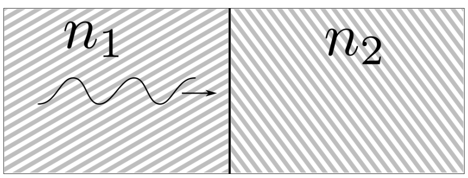
Problem 132. *1989-Spring-EM-G-5* ID:EM-G-80

Answer the following questions concerning the propagation of electromagnetic radiation through a nonmagnetic, dielectric media.

1. From Maxwell's equations, derive the boundary conditions for the normal and tangential components of the $\vec{E}$ and $\vec{B}$ fields at the interface between two non-magnetic, dielectric materials.

2. For a plane, linearly polarized wave of normal incidence from the left as shown determine the conditions satisfied by the electric field amplitudes of the incident E_i, reflected E_r, and transmitted E_t waves. (Note that for a plane wave propagating through a non-magnetic medium of index of refraction n, the magnitudes of $\vec{E}$ and $\vec{B}$ are related by $E = \frac{c}{n}B$

3. Find the ratio of the reflected *intensity* to the incident *intensity* at the interface shown.



Problem 133. *1990-Fall-EM-G-4* ID:EM-G-83

Consider an infinite sheet in $x - z$ plane of thickness $2L$ in the y-direction (from $y = -L$ to $y = L$), carrying a uniform current per unit area J in the x-direction.

1. Find the magnetic field, in both magnitude and direction, both above and below the sheet.

2. Find the magnetic field, in both magnitude and direction, within the sheet.

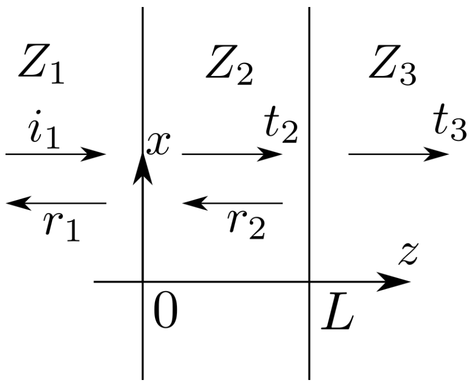
3. If $dJ/dt = 2 \times 10^7 \mathrm{A/m^2s}$, what is the maximum induced emf for a square circuit of area $2\mathrm{cm}^2$ placed outside a 2mm thick sheet? How should the circuit be oriented to get the maximum emf?

Problem 134. *1990-Fall-EM-G-5.jpg* ID:EM-G-86

Consider a monochromatic, linearly polarized plane wave that is normally incident on the interface between two non-magnetic dielectrics in the $z = 0$ plane. These two dielectrics have *optical impedances* of Z_1 and Z_2 respectively, where the optical impedance is defined as $Z_i = \sqrt{\mu_i/\epsilon_i}$. The second medium, Z_2, has a thickness L, and behind it is another medium of optical impedance Z_3 which fills the test of space to the right of this interface. In the figure you are shown the incident, reflected and transmitted waves that you are to consider in each of the three regions above.

1. If the electric field in the incident wave can be represented as $E_{1i}e^{i(k_1 z - \omega t)}\hat{x}$, write out similar expressions for the $\vec{E}$ and $\vec{B}$ fields in all three regions shown. Label your fields clearly!

2. State the boundary conditions which the fields must satisfy at $z = 0$ and $z = L$?
3. Find the ratio E_{1r}/E_{1i}.
4. If $Z_1 \neq Z_3$, find the condition on the thickness of the coating, L, and the value of Z_2 which makes the ratio in the previous part equal 0 (e.g. no reflection).

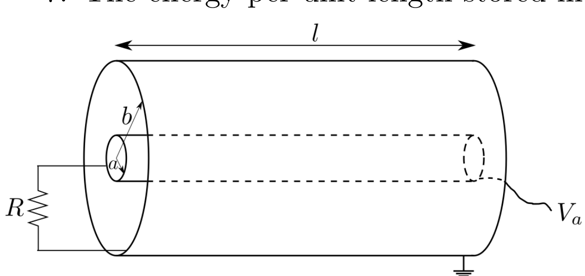


Problem 135. *1990-Spring-EM-G-4* ID:EM-G-89

Consider a cylindrical coaxial cable of length l, consisting of a thin outer conductor of radius b and inner conductor of radius a. Assume that the resistance of the conductors is negligible, that no dielectric is present and that $l \gg b$, so that end effects can be neglected. At one end, the outer conductor is grounded and the inner conductor is maintained at potential $+V_a$, and at the other end the circuit is completed by connecting a resistance R between the inner and outer conductors.

In terms of the quantities given, find expressions for the following:

1. The electric field $\vec{E}$ for all r.
2. The magnetic induction $\vec{B}$ for all r.
3. The electric potential V_r for all r.
4. The capacitance per unit length of the cable.
5. The self-inductance per unit length of the cable.
6. The energy per unit length stored in the E-field.
7. The energy per unit length stored in the B-field.



Problem 136. *1990-Spring-EM-G-5* ID:EM-G-92

A plane electromagnetic wave in a medium of refractive index n_1 , is incident at an angle θ_i with respect to the normal of a second medium of index of refraction n_2. Assume the electric fields in these waves are perpendicular to the plane of incidence as shown in the figure. The electric fields for the incident, reflected, and refracted waves can be written as follows:

$$E_z = E_0 e^{-i\omega t + i\vec{k}\cdot\vec{r}}$$
$$E'_z = E'_0 e^{-i\omega t + i\vec{k}'\cdot\vec{r}}$$
$$E''_z = E''_0 e^{-i\omega t + i\vec{k}''\cdot\vec{r}},$$

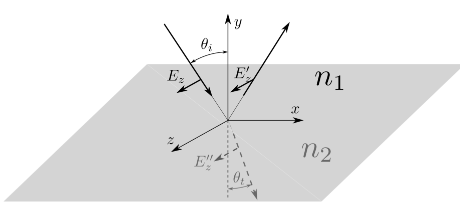
where $|\vec{k}| = |\vec{k}'| = n_1\omega/c$, $|\vec{k}''| = n_2\omega/c$. Assume that $\mu_1 = \mu_2 = \mu_0$.

1. Use the boundary conditions to determine the reflection coefficient r, where $r = E'_0/E_0$. Express your answer in terms of n_1, n_2, and θ_i. Hint: Use Snell's law to eliminate θ_t.

2. If $n_1 > n_2$, use this result to determine the critical angle θ_c, for total internal reflection.

3. Determine the magnitude of r when $\theta_i > \theta_c$.

4. Determine the phase shift δ, upon reflection when $\theta_i > \theta_c$. It is okay to leave your answer in the form $\tan(\delta) = \ldots$.



Problem 137. *1991-Fall-EM-G-4* ID:EM-G-95

1. There are two infinite planes perpendicular to the z-axis and located at $z = a/2$ and $z = -a/2$. The space between the planes is filled with a uniform positive volume charge density ρ. Determine the potential everywhere. Take the potential at $z = 0$ to be zero.

2. An infinite plane at $z = 0$ has a periodically varying areal charge density given by
$$\sigma = \sigma_0 \sin(\alpha x) \sin(\beta y)$$
where σ_0, α, and β are constants. Determine the potential ϕ everywhere in space. Take the potential at the point $(0, 0, 0)$ to be zero.

Problem 138. *1991-Fall-EM-G-5* ID:EM-G-98

An electromagnetic plane wave in free space has an angular frequency ω and propagation vector $\vec{k}$ measured in an inertial coordinate system x, y, z, t.

1. If $\vec{k} = k\hat{x}$, find ω' and $\vec{k}'$, the corresponding quantities measured in a Lorentz-transformed coordinate system x', y', z', t' moving with a uniform velocity $\vec{v} = v\hat{x}$.

2. Find ω' and $\vec{k}'$ as in the previous part except that the direction of $\vec{k}$ bisects the x and y axes.

3. A plane wave directed as in the previous part is reflected from a perfectly reflecting flat mirror perpendicular to the x axis and moving with a velocity $\vec{v} = v\hat{x}$ Find ω_r and $\vec{k}_r$, the frequency and propagation vector of the reflected plane wave. (Assume that $\vec{v}$ is such that reflection can occur.)

Problem 139. *1991-Spring-EM-G-4* ID:EM-G-101

There is a uniform magnetic field in the $+z$ direction, $B_0\hat{k}$. An electron moves in a circular orbit of radius r_0 in the $x-y$ plane with angular velocity ω_0. The field is now increased linearly to a value B_f in a time τ.

1. Draw a figure indicating the direction of B and of the circular orbit.

2. Determine ω_f.

3. The electron's energy has increased. Determine the force that increases the electron's velocity. Redraw the figure of the previous part and indicate the direction of this force at several points.

4. Use this force to determine the infinitesimal change in speed dv due to a change dB in the field. Also determine dv using the centripetal force relation. Use these two results to determine a relation between r and B.

5. Determine r_f and v_f.

6. How is the initial flux Φ_0 through the orbit related to the final flux Φ_f?

   In a model of the atom, an electron moves in a circular orbit of radius r_0 about the nucleus, in the $x-y$ plane, with angular velocity ω_0. The only force is the Coulomb attraction (no magnetic field). It can move either clockwise or counter-clockwise.

   A uniform magnetic field, $B_0\hat{k}$, is now applied in the $+z$ direction, being increased linearly in a time τ from zero to B_0. The angular velocity changes by a small amount $\Delta\omega$. Assume the centripetal force due to the Coulomb attraction is much greater than that due to the magnetic field, so that r_0 does not change.

7. Determine $\Delta\omega$ for each direction of rotation.

8. The magnetic moments of two such oppositely circulating electrons in an atom cancel when the magnetic field is zero but do not cancel when a magnetic field is applied. Determine the resultant magnetic moment, and indicate its direction with respect to the applied field.

9. A medium made up of such atoms is said to be which of the following: diamagnetic, paramagnetic, ferromagnetic, superconducting?

Problem 140. *1991-Spring-EM-G-5* ID:EM-G-104

A conducting ring of radius a, carrying a total charge Q, is at a distance d above an infinite conducting plate, which is maintained at a potential V.

1. Show that the electrostatic potential $\Phi(\vec{r})$ satisfies the Laplace equation in the charge-free region of the space above the conducting plate.

2. Write down the most general series solution for $\Phi(\vec{r})$ in the charge-free region, based on special solutions of the Laplace equation in spherical coordinates.

3. Find $\Phi(\vec{r})$ along the axis of the ring by another method, and use it to find the unknown coefficients of the series solution of the previous part. Your answer is actually different, depending on whether $r > \sqrt{a^2+d^2}$ or $r < \sqrt{a^2+d^2}$. Explain why.

4. Find series solutions to the induced surface charge distribution on the conducting plate. Again, you should have two different answers, depending on whether $r > \sqrt{a^2+d^2}$ or $r < \sqrt{a^2+d^2}$.

   Useful formula:
   $$\frac{1}{|\vec{x}-\vec{x}'|} = \sum_{l=0}^{\infty} \frac{r_<^l}{r_>^{l+1}} P_l(\cos\gamma),$$
   where γ is the angle between $\vec{x}$ and $\vec{x}'$.

Problem 141. *1992-Fall-EM-G-4* ID:EM-G-107

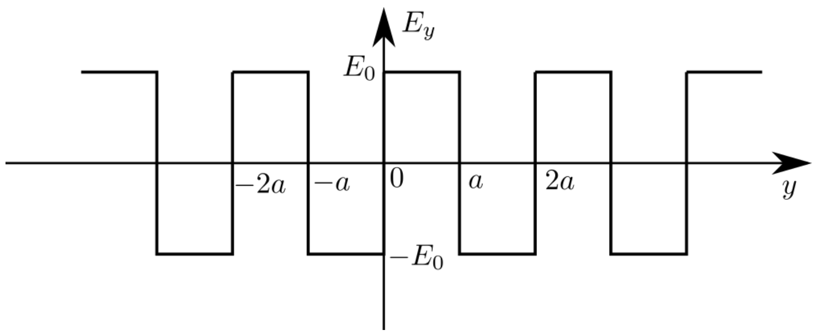
Consider an infinite sheet of charge defined by the plane $x = 0$. It is given that at $x = 0$, the $y$component of the electric field generated by this charge distribution, E_y, is a periodic function of y with period $2a$, taking the form shown on the figure, and is independent of z.

1. Find the potential $\Phi(x, y, z)$ for $x > 0$.

2. Find E_x for $x > 0$.

3. Find the areal charge density $\sigma(y, z)$ on $x = 0$.

4. Sketch the charge density from $y = -a$ to $y = a$.



Problem 142. *1992-Fall-EM-G-5* ID:EM-G-110

1. A uniformly charged disk of areal charge density σ exists in the region $\rho \leq a$, $z = 0$, in cylindrical coordinates (ρ, ϕ, z). Calculate the electrostatic potential $\Phi(z)$ on the z-axis for both $z > 0$ and $z < 0$

2. Expand the above $\Phi(z)$ in powers of $a/|z|$ in the region $|z| > a$, to at least three non-vanishing terms, and use this expansion to obtain the solution for $\Phi(r, \theta)$ in the region $r > a$, $0 \leq \theta \leq \pi$, in *spherical coordinates* (r, θ, ϕ), in the form of an expansion involving $P_l(\cos(\theta))$ to at least $l = 4$.

   [Given:

   (a) The Laplace operator in spherical coordinates is:
   $$\nabla^2\psi = \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2}\left[\frac{1}{\sin(\theta)}\frac{\partial}{\partial\theta}\left(\sin(\theta)\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{\sin^2(\theta)}\frac{\partial^2\psi}{\partial\phi^2}\right].$$

   (b)
   $$\frac{1}{\sin(\theta)}\frac{\partial}{\partial\theta}\left[\sin(\theta)\frac{\partial}{\partial\theta}P_l(\cos(\theta))\right] + l(l+1)P_l(\cos(\theta)) = 0.$$

   (c)
   $$P_l(1) = 1, \quad \text{for all } l = 0, 1, 2, \ldots.$$

   ]

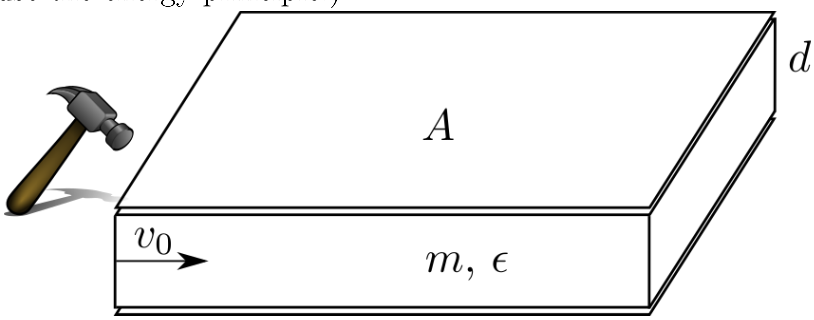
Problem 143. *1992-Spring-EM-G-4* ID:EM-G-113

A rectangular dielectric block of mass M, dielectric constant ϵ, thickness d and cross sectional area A, is initially completely inside a parallel plate capacitor of area A and separation slightly bigger than d. A sudden impulse is then given to the dielectric block, which gives the block an initial velocity v_0 in the direction shown. Calculate the minimum value of v_0 such that the block can leave the capacitor, for both of the following two situations:

1. The charge Q on the capacitor is kept constant.

2. The voltage V across the capacitor is kept constant.

Neglect edge effect, friction, and gravity. (Hint: One method to solve this problem is to use the energy principle.)

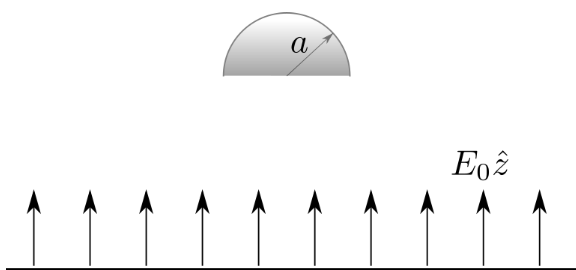


Problem 144. *1992-Spring-EM-G-5* ID:EM-G-116

An uncharged conducting hemisphere of radius a is initially infinitely far from a grounded conducting plane which has a normal electric field $E = E_0\hat{z}$. The hemisphere is then brought into flat contact with the plane, allowed to reach equilibrium, and then removed to infinity. In being removed from the plane, all points of the flat surface leave the plane at the same time.

1. What is the charge on the hemisphere after it is removed?

2. What is the electric field at the conducting plane after the hemisphere is removed?



Problem 145. *1993-Fall-EM-G-4* ID:EM-G-119

1. Using the Biot-Savart Law $d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\vec{r}}{r^3}$ find the magnetic field at a point P along the normal to the center at distance x to the center of the current I carrying loop.

2. Find the force on a magnetic monopole q^* placed at P.

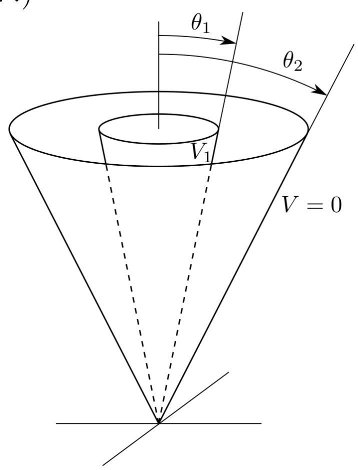
3. Find the magnetic field produced by this monopole q^*, at the position of the loop. (The Coulomb's law constant $\frac{1}{4\pi\epsilon_0}$ is replaced by $\frac{\mu_0}{4\pi}$ for magnetic monopole.

4. Using the result of the previous part calculate the force on the loop due to the monopole.

Problem 146. *1993-Fall-EM-G-5* ID:EM-G-122

Solve Laplace's equation for the electrostatic potential in the region between two infinite coaxial cones, as shown in the diagram. The cone at θ_1 , has potential V_1, and the cone at θ_2 has potential $V = 0$. The cone vertices are insulated at $r = 0$.

(HINT: You may neglect edge effects, and assume that the solution is independent of r.)



Problem 147. *1993-Spring-EM-G-4* ID:EM-G-125

1. A slab of uniform current density $\vec{J} = J_0\hat{e}_y$, has boundaries at $z = -b/2$ and $z = b/2$. Find the magnetic field both inside and outside the slab.

2. A neutron, moving at a velocity $\vec{v} = v_0\hat{e}_z$, passes through this slab. If the magnetic dipole moment of the neutron is modeled by a small square current loop with moment $\vec{m} = m_0\hat{e}_z$, and the loop edges parallel to the y and z directions, find the total momentum transfer to the neutron. Take $\vec{v}$ and $\vec{m}$ to be constant during the passage through the slab.

   [Note: Recall that the magnetic moment of a current loop is equal to its current I times the area of the loop a^2. Here the limit $a \to 0$ and $I \to \infty$ should be taken such that $Ia^2 = m_0$ remains finite.]

3. Repeat the previous part if the magnetic dipole moment of the neutron is due to a pair of magnetic monopoles of strengths $+q^*$ and $-q^*$, respectively, separated by a distance d in the x direction, such that $q^*d = m_0$.

4. Show that the force on the neutron is zero for $z > b/2$ for both models of the neutron.

Problem 148. *1993-Spring-EM-G-5* ID:EM-G-128

Consider an infinite block of linear, isotropic, homogeneous, conducting material of conductivity σ_0. (There is no electric susceptibility in this material.) In the center of this block and oriented parallel to the z-axis is an infinitely long cylindrical cavity of radius a, in which no charge exists. Very far from the cavity at the edges of this infinite block of material, we observe that there is a uniform current density, $\vec{J} = j_0 \hat{e}_x$, where J_0 is a constant.

1. Give a simple proof that in steady state there can be no bulk charge distribution inside the conducting material.

2. What equation governs the electrostatic scalar potential Φ inside the material? What is the most general solution of this equation in cylindrical coordinates, taking into account the translational symmetry of this problem along z?

3. What are the boundary conditions for Φ very far from the cylindrical cavity, and on the surface of this cavity? Use these boundary conditions to fix all unknown constants in the general solution for Φ that you obtained in the previous part.

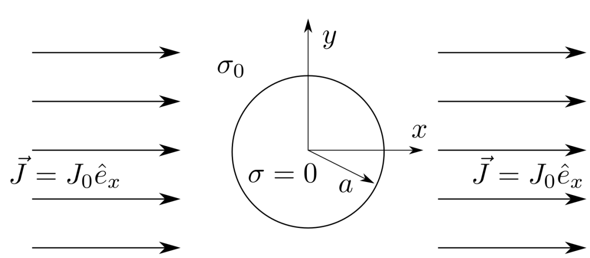
4. Using this solution, determine the steady-state current distribution $\vec{J}(\rho, \phi)$ everywhere inside the conducting material.

5. Show that there must also be a surface charge distribution existing on the surface of the cylindrical cavity, and obtain this distribution.

[Hint: First show that the electric field distribution inside the cavity must he a constant, and then use suitable boundary conditions to determine this constant field. Finally, relate this field to the surface charge density.)

[Given: In cylindrical coordinates (ρ, ϕ, z)

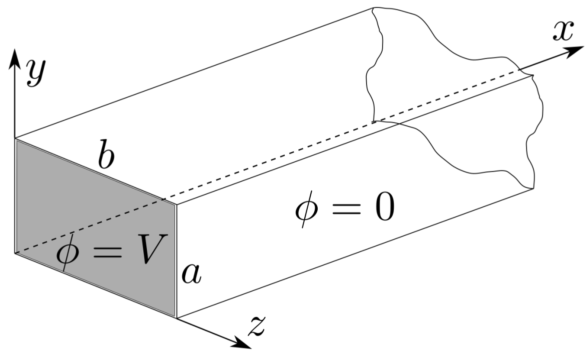
$$\nabla u = \hat{e}_\rho \frac{\partial u}{\partial \rho} + \hat{e}_\phi \frac{1}{\rho} \frac{\partial u}{\partial \phi} + \hat{e}_z \frac{\partial u}{\partial z}$$
$$\nabla \cdot \vec{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$
$$\nabla^2 u = \frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho \frac{\partial u}{\partial \rho}\right) + \frac{1}{\rho^2} \frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2}$$



Problem 149. *1994-Fall-EM-G-4* ID:EM-G-131

A semi-infinite metal pipe extends from $x = 0$ to $x = \infty$. It has rectangular cross-section with sides of length a (along y) and b (along z). The pipe is grounded, and the face at $x = 0$ is insulated from the rest of the pipe and is held at potential V.

Calculate the potential $\phi(x, y, z)$ everywhere inside the pipe.



Problem 150. *1994-Fall-EM-G-5* ID:EM-G-134

An electromagnetic plane wave of intensity I ($I \equiv \langle S \rangle$, $\vec{S} = \frac{1}{\mu}\vec{E} \times \vec{B}$) propagating in a vacuum falls upon a glass plane with index of refraction n. The wave vector $\vec{k}$ is incident normally on the surface of the glass. The electric and magnetic fields can be written $\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$; $\vec{B} = \vec{B}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} = \frac{1}{v}\hat{k} \times \vec{E}$. Assume $\mu = \mu_0$ in both the glass and the vacuum.

1. Write down the boundary conditions relating $\vec{E}$, $\vec{D}$, $\vec{B}$, and $\vec{H}$ on the two sides of the surface in general terms.

2. For each boundary condition in the previous part part, write down the appropriate relation for this particular problem.

3. Solve the relation from the previous part for the ratio $E_{\text{refracted}}/E_{\text{incident}}$.

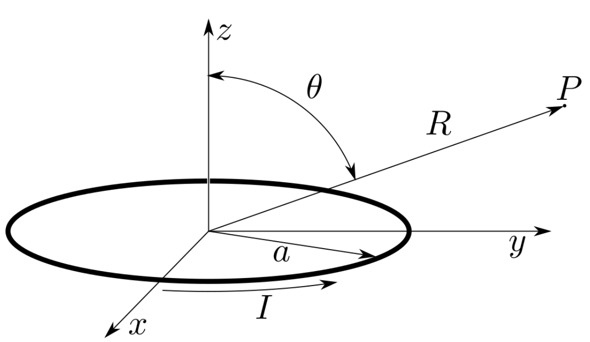
4. Show that the coefficient of reflection (of the intensity) is given by $R = \frac{(n-1)^2}{(n+1)^2}$.

Problem 151. *1994-Spring-EM-G-4* ID:EM-G-137

Consider a small circular loop of radius a that carries a current I.

1. Calculate the vector magnetic potential at point P shown in figure. You can approximate your result by taking $R \gg a$.

2. Calculate the magnetic field at point P.

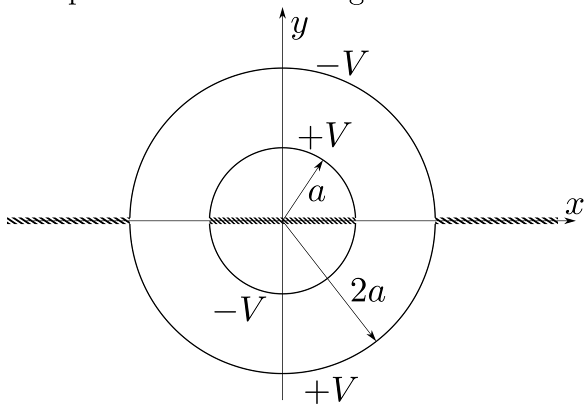


Problem 152. *1994-Spring-EM-G-5* ID:EM-G-140

A grounded conducting sphere of radius a is placed in an initially uniform electric field E_0 directed along the z axis. Calculate the electrostatic potential everywhere outside the sphere. Calculate the electric field outside the sphere, and explain the significance of each term in your expression.

Problem 153. *1995-Fall-EM-G-1* ID:EM-G-143

A pair of infinite, coaxial cylinders have radii a and $2a$. The cylinders consist of conductors, except for three very thin insulators in the plane $y = 0$. The resultant half-cylinders are held at potentials $\pm V$, as indicated in the figure. What is the electric potential Φ in the region between the two cylinders?



Problem 154. *1995-Fall-EM-G-2* ID:EM-G-146

Consider a one-resonant-frequency model for the dielectric constant of a material

$$\epsilon(\omega) = 1 + \frac{\omega_p}{\omega_0^2 - \omega^2 - i\omega\gamma} \equiv 1 + i\frac{4\pi\sigma(\omega)}{\omega},$$

where ω_0 is the resonant frequency, γ is a damping constant, ω_p is the plasma frequency, and σ is the complex conductivity.

1. What effect does $\Im\epsilon(\omega)$ have on wave propagation?
2. What happens to waves incident on free electron plasmas for $\omega < \omega_p$? Explain.
3. Obtain an expression (in terms of σ) for the "skin depth" δ in a good conductor.

Problem 155. *1995-Fall-EM-G-3* ID:EM-G-149

Maxwell's equations in the presence of a charge and current density $\rho(\vec{r}, t)$ and $\vec{J}(\vec{r}, t)$ read:

$$\vec{\nabla} \cdot \vec{E} = \tfrac{4\pi}{\epsilon_0}\rho, \qquad \vec{\nabla} \times \vec{B} = \frac{4\pi}{\mu_0 c}\vec{J} + \frac{\epsilon_0}{\mu_0 c}\frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0, \qquad \vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$$

In terms of potentials $\vec{A}$ and ϕ

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}, \qquad \vec{B} = \vec{\nabla} \times \vec{A}.$$

1. Show that:

$$\nabla^2\phi + \frac{1}{c}\frac{\partial}{\partial t}\left(\vec{\nabla}\cdot\vec{A}\right) = -4\pi\rho$$
$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} - \vec{\nabla}\left(\vec{\nabla}\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right) = -\frac{4\pi}{c}\vec{J}$$

2. Define the gauge transformation of $\vec{A}$ and ϕ.

3. The Lorentz gauge is defined by:

$$\vec{\nabla}\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t} = 0.$$

   Show that starting in an arbitrary gauge there is always a gauge function Λ that allows one to set up the Lorentz gauge. What is the general class of Λ that preserve the Lorentz gauge conditions?

4. The axial gauge is defined by $A_z(\vec{r}, t) = 0$. Show that the axial gauge can always be set up and find the general class of Λ that preserve the axial gauge.

Problem 156. *1995-Spring-EM-G-1* ID:EM-G-152

A slab of insulating material of thickness l is between two conducting plates one of which is transparent. A battery of emf V_b produces a potential drop across the material and there is a resistor R in series in order to measure the current. A pulse of light produces a positive charge at the transparent plate which is swept across the thickness of the insulator with a constant mobility μ (defined as the velocity per electric field strength). Assume that this photocharge has an areal density σ and has drifted to a position x_0 between the plates.

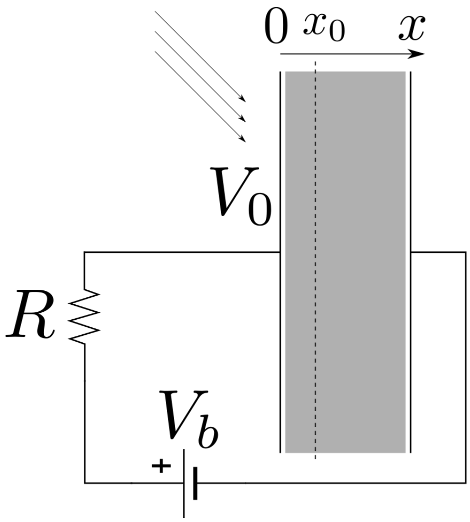
1. State the requirements (differential equations and boundary conditions) upon the electric potential for all points in the insulating material from these photocharges and the potential applied to the plates.

2. Find the equation for the electric potential which meets these requirements.

3. From this equation, find the charge on one of the conducting plates as a function of the position of the layer of charge within the insulator.

4. From this charge, find the current which passes through the resistor as this charge moves between the plates.

5. If instead of a pulse of light, the light is turned on and left on, describe qualitatively the current through the resistor. Do not do further calculations.



Problem 157. *1995-Spring-EM-G-2* ID:EM-G-155

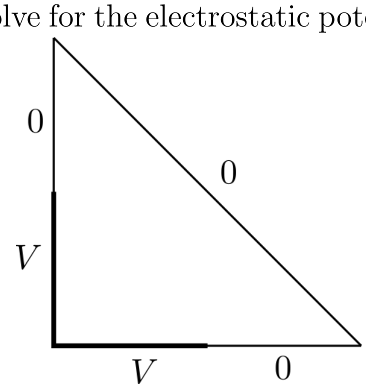
An infinite hollow tube has a uniform cross section in the shape of a right triangle with sides of length a, a, $\sqrt{2}a$. The sides of length a are on the x- and y-axes, as shown. The electrostatic potential Φ on the surfaces is:

x-axis: $\Phi(x, y = 0) = +V$, for $x < a/2$, and $\Phi = 0$ otherwise,

y-axis: $\Phi(x = 0, y) = +V$, for $y < a/2$, and $\Phi = 0$ otherwise,

diagonal: $\Phi(x, y = a - x) = 0$, for all $0 \leq x \leq a$.

Solve for the electrostatic potential $\Phi(x, y)$ throughout the interior of the hollow tube.

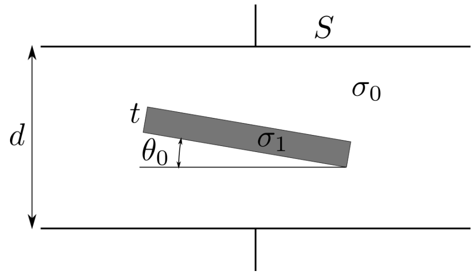


Problem 158. *1995-Spring-EM-G-3* ID:EM-G-158

A space between two parallel plates (area S, separation d) is filled by a liquid having a conductivity σ_0. A resistive plate (thickness t, conductivity σ_1) is placed in the liquid at an angle θ_0 as shown in the figure. Assume θ_0 is small and that the change in the electric field in the liquid (E_0) is negligible when the plate is inserted. Neglect end effects.

1. What are the boundary conditions for $E_\perp$ and $E_\parallel$ on the surfaces of the resistive plate.

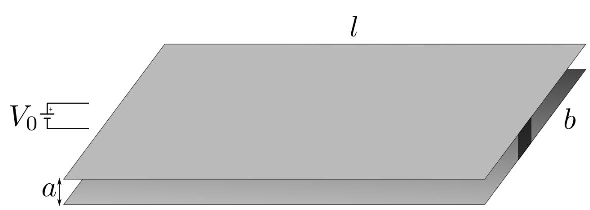
2. Find the total resistance between the parallel plates. (Given $\cos(\theta) \approx 1 - \frac{1}{2}\theta^2$ for small θ.)



Problem 159. *1996-Fall-EM-G-4* ID:EM-G-161

The system shown in the diagram consists of two flat conducting strips, each of length l, width b (perpendicular to the plane of the diagram), and uniformly distributed resistance R, separated by a small gap a ($q \ll l, b$). The right ends of the strips are shorted, and the battery voltage V_a is connected across the left ends at time $t = 0$. The current is assumed to flow only parallel to the l-dimension of the strips. Neglect all effects arising from the finite speed of propagation of electro-magnetic fields.

1. What is the relation between the magnetic field B between the strips and the current I flowing in the circuit?
2. What is the self inductance of the circuit?
3. What is the current in the circuit as a function of time?
4. What is the voltage across the strips as a function of the distance x from the shorted end?
5. At a time $t = 0^+$ (just after the battery is connected), what is the flow of energy down the system as a function of distance from the shorted end? Describe where the energy goes.

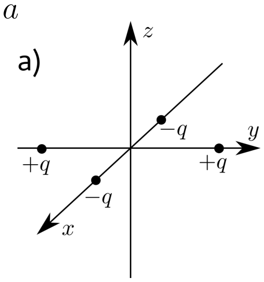


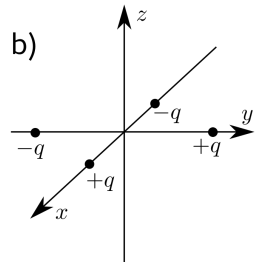
Problem 160. *1996-Fall-EM-G-6* ID:EM-G-167

Consider a dielectric sphere of radius R and dielectric constant ϵ placed inside a homogeneous electric field $\vec{E} = E_0\hat{e}_z$. Calculate the electric field $\vec{E}$ and the potential ϕ inside and outside the sphere. How large is the induced dipole moment?

Problem 161. *1996-Spring-EM-G-1* ID:EM-G-170

Calculate the dipole and quadrupole tensor moments of the following two charge distributions. Assume that the distance of the charges from the origin is in each case is a





Problem 162. *1996-Spring-EM-G-2* ID:EM-G-173

1. Given the Maxwell equations in vacuum

$$\vec{\nabla}\cdot\vec{E}=0, \qquad \vec{\nabla}\times\vec{B}-\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}=0$$

$$\vec{\nabla}\cdot\vec{B}=0, \qquad \vec{\nabla}\times\vec{E}+\frac{\partial\vec{B}}{\partial t}=0$$

   Derive the wave equation in free space.

2. Write the Maxwell equations for a dilute lossy medium having conductivity σ, and show that for plane electromagnetic waves passing through such a medium we have

$$-\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right)=-\mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2}+\mu_0\sigma\frac{\partial\vec{E}}{\partial t}$$

   Show that this leads to a damped wave.

Problem 163. *1996-Spring-EM-G-3* ID:EM-G-176

A permanent magnet consists of an infinitely long cylinder, of radius R, in which there is a "frozen-in" magnetization $\vec{M}=kr\hat{z}$, where k is a constant, r is the distance from the axis, and $\hat{z}$ is a unit vector parallel to the axis. There are no free currents anywhere. Calculate $\vec{A}$, $\vec{H}$, $\vec{B}$, and the distribution of bound currents for locations inside and outside the magnet.

Problem 164. *1997-Fall-EM-G-4* ID:EM-G-179

Consider a region of space filled with N *non-interacting* electrons per cubic meter. The conductivity of this gas of electrons is

$$\sigma(\omega)=i\left(\frac{Ne^2}{\omega m}\right),$$

where e is the electron charge and m is the electron mass.

1. From Maxwell's equations, find the velocity of propagation of electro-magnetic waves in this space.

2. Find the index of refraction of this region.

3. In the ionosphere the density of electrons is approximately 10^{11} electrons/m^3. Over what frequency range will electromagnetic waves be allowed to propagate in this region?

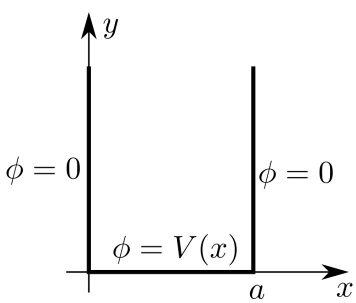
4. Derive the expression for the conductivity given in the problem statement above when the electrons are subject to a time dependent electric field of the form $E(t) = E_0 e^{-i\omega t}$.

Problem 165. *1997-Fall-EM-G-5* ID:EM-G-182

Consider a 2-dimensional electrostatic problem, in which an semi-infinite channel, closed at the bottom and open at the top, has potentials set up as shown in the figure.

1. Calculate the potential everywhere inside the channel, in the case where $V(x) = V_0 \sin(n\pi x/a)$.

2. Calculate the total electrostatic energy, $W = \frac{1}{8\pi}\int E^2 dxdy$ inside the channel.



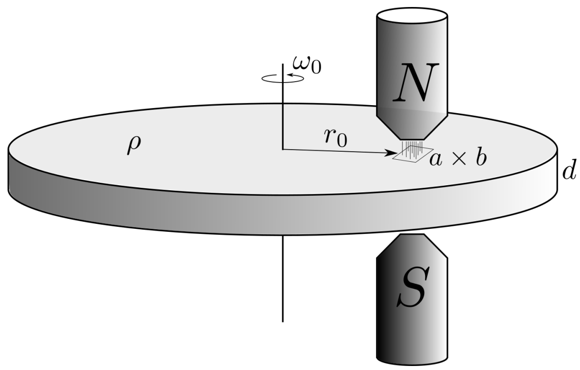
Problem 166. *1997-Spring-EM-G-4.jpg* ID:EM-G-185

A conducting disk of radius R_0, resistivity ρ, and thickness d, is spinning with an angular velocity ω_0 about a vertical axis. A small rectangular region of dimension a in the tangential direction and b in the radial direction is subject to a uniform vertically downward magnetic field B_0. This region has its inner edge at a distance of r_0 from the axis of rotation such that $a \ll b \ll r_0$.

1. Determine the electrical resistance R of this region for both radial and tangential currents.

2. Determine the current flowing through this region, including its direction.

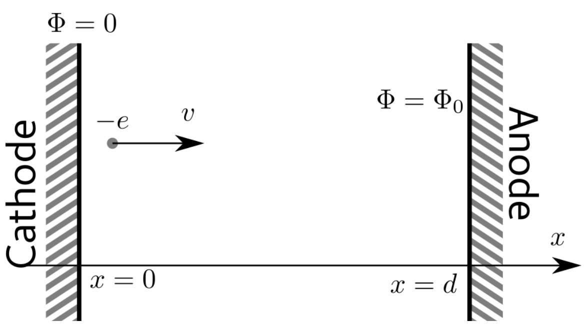
3. Calculate the torque on the disk due to this current.



Problem 167. *1997-Spring-EM-G-5.jpg* ID:EM-G-188

Consider a parallel plate *thermionic vacuum tube* which is idealized as a one- dimensional structure for this problem. A constant potential difference, $\Phi_0 > 0$, is maintained between the heated cathode and the anode, which are separated by a distance, d. Electrons of charge $-e$ are liberated thermionically in unlimited quantities at the cathode and are accelerated to the anode. Consider that the electrons are liberated at the cathode surface at $x = 0$ with **zero velocity**. The region between the plates is a vacuum except for the copious number of electrons that are emitted into it. These electrons constitute a space charge with a charge density, $\rho = \rho(x)$ once equilibrium is reached. In equilibrium, the electrons depress the electric field in the region very near the cathode in such a way that, $E = 0$ at $x = 0$.

1. Explain why $\vec{J}$, the current density between the plates, is independent of x in equilibrium.
2. In the limit of no collisions between the electrons, find the velocity of these electrons as a function of their x position.
3. Find the charge density $\rho(x)$.
4. Find the value of the potential between the plates $\Phi(x)$.
5. Determine the how the current density $\vec{J}$, depends on the value of Φ_0,



Problem 168. *1998-Fall-EM-G-4* ID:EM-G-191

Consider an electromagnetic wave in an uncharged medium with permittivity ϵ, magnetic permeability μ and conductivity κ.

1. Write Maxwell's equations when both conducting and displacement currents can not be ignored.

2. Find the differential equation for $\vec{E}(z,t)$ from the previous part for an electromagnetic wave traveling in the z-direction.

3. If $\vec{E}$ oscillates in time as:
$$\vec{E}(z,t) = \vec{E}(z)e^{i\omega t}\hat{x}$$
and propagates in the $+z$ direction, find $E(z)$. Assume $|\kappa/\epsilon\omega| \ll 1$.

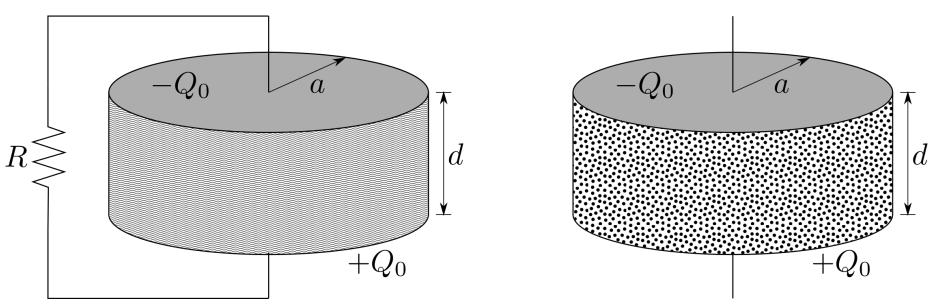
4. Find the distance over which $\vec{E}||$ decreases by a factor of e ($e = 2.718$).

Problem 169. *1998-Fall-EM-G-5* ID:EM-G-194

You are given two parallel plate capacitors both of the same capacitance and both containing the same total charge Q, at $t = 0$. Both capacitors are made of thin circular plates of conductor of radius a and with a plate separation d. Capacitor 1 is filled with an *insulating* material with a dielectric constant κ and is connected to an infinitely long straight wire of resistance R that connects the two terminals of the capacitor together. Capacitor 2 is filled with a slightly conducting dielectric of conductivity σ and dielectric constant κ. At $t = 0$ both capacitors begin to discharge.

1. Find the magnetic field produced inside the volume of the capacitors for **each of these cases** as a function of time.

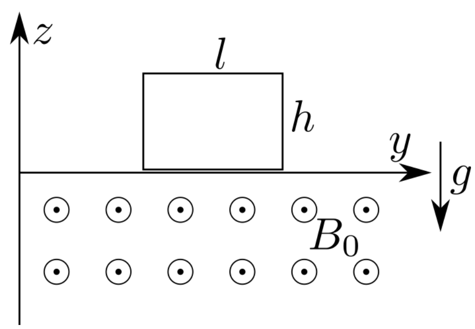
2. Calculate the rate of energy flow into or out of the capacitor volume for these **two cases** as a function of time.



Problem 170. *1998-Spring-EM-G-4* ID:EM-G-197

You are given a loop of superconducting wire of dimension l by h, self-inductance L and mass m. This loop is parallel to the $y-z$ plane with its lower edge at the boundary of a region of uniform B-field where:
$$\vec{B} = 0, \quad \text{for } z > 0, \qquad \vec{B} = B_0\hat{x}, \quad \text{for } z < 0$$
At $t = 0$ the loop is released from rest and constrained to remain in the $y-z$ plane. (Gravity acts along the negative z direction.)

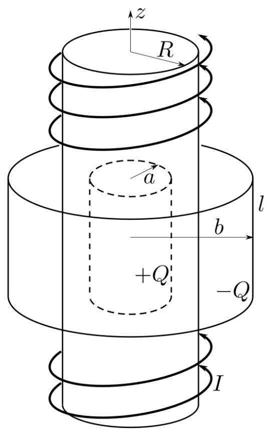
1. Set up the relationship between the induced emf and the velocity of the falling loop of wire after it is released.
2. Set up the equations necessary to solve for the vertical position of the loop as a function of time.
3. Using the results from the previous parts find the resulting current and vertical position of the wire loop as a function of time.
4. The motion of the loop is characteristically different at high and low magnetic fields. Find the critical value of the magnetic field separating these two regimes.



Problem 171. *1998-Spring-EM-G-5* ID:EM-G-200

A very long solenoid with radius R and N turns per unit length has a current I. Coaxial with the solenoid are two long cylindrical shells of length l, that are free to rotate; one inside the solenoid at a radius a with total charge $+Q$ and one outside the solenoid at a radius b with total charge $-Q$. The charge is rigidly fixed to the cylindrical shells and is uniformly distributed over their surfaces.

1. Determine the electric and magnetic fields for this system. (Be sure to specify the magnitude, direction, functional dependence and location of each of the fields.) One can assume that the length $l \gg a, b$ so that edge effects can be ignored.
2. If the current in the solenoid is gradually reduced to zero in a time τ, the charged cylinders are observed to rotate. Determine the final angular momentum of each of the charged cylinders.
3. Show that the angular momentum gained by the charged cylinders is equal to the angular momentum in the fields *before* the current was reduced.



Problem 172. *1999-Fall-EM-G-4* ID:EM-G-203

A grounded conducting sphere of radius a is placed in an initially uniform electric field E_0 directed along the z-axis. Calculate the electrostatic potential everywhere outside the sphere. Calculate the electric field outside the sphere, and interpret each term in your expression.

Note: The first few Legendre polynomials are $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = (3x^2 - 1)/2$. The gradient and Laplacian operators in spherical polar coordinates are

$$\vec{\nabla} = \hat{e}_r \frac{\partial}{\partial r} + \hat{e}_\theta \frac{1}{r}\frac{\partial}{\partial \theta} + \hat{e}_\phi \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \phi}$$
$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2\sin^2(\theta)}\frac{\partial^2}{\partial \phi^2}.$$

Problem 173. *1999-Fall-EM-G-5* ID:EM-G-206

Maxwell's equations are

$$\vec{\nabla}\cdot\vec{E} = \tfrac{1}{\epsilon_0}\rho. \qquad \vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\frac{\partial \vec{E}}{\partial t} + \mu_0\vec{J}$$
$$\vec{\nabla}\cdot\vec{B} = 0. \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

1. Show that in free space, with no charges or currents, the equations admit plane electromagnetic wave solutions. Obtain the dispersion relation that relates the magnitude of the wave-vector $\vec{k}$ and the angular frequency ω for plane waves. Calculate the speed of the waves, in terms of μ_0 and ϵ_0.

2. Suppose now that there is a conducting medium, with (constant) conductivity σ, such $\vec{J} = \sigma\vec{E}$. Show that the wave equation now acquires a damping term, and derive the dispersion relation for this case. Obtain an approximate leading-order expression for the "skin depth," where you may assume that σ is sufficiently large as to simplify the algebraic expression that you derive. (The skin depth is defined to be the distance over which the amplitude of the wave decays by a factor of e.)

Problem 174. *1999-Spring-EM-G-4* ID:EM-G-209

1. Write down the Lorentz transformation of the coordinates (t, x, y, z) for a boost with velocity $\vec{v} = (0, 0, v)$ directed along the z-axis.

2. A particle with charge q moves with constant velocity $\vec{v} = (0, 0, v)$ along the z-axis in the laboratory frame. By making a Lorentz transformation from the rest frame of the particle, show that in the laboratory frame the potentials are

$$\phi = \frac{q}{s}, \qquad \vec{A} = \frac{\vec{v}}{c}\phi,$$

where $s = \left[(1 - v^2/c^2)(x^2 + y^2) + (z - vt)^2\right]^{1/2}$ (Recall that $(\phi, \vec{A})$ transform like $(ct, \vec{r})$ under Lorentz transformations.)

3. Calculate the fields $\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$ and $\vec{B} = \vec{\nabla} \times \vec{A}$ at the point (x, y, z) at time t. (Hint: First show that $\vec{B} = \vec{v} \times \vec{E}/c$, and then calculate $\vec{E}$ explicitly.) Show that $\vec{E}$ is parallel to $\vec{R} \equiv (x, y, z - vt)$.

Problem 175. *1999-Spring-EM-G-5* ID:EM-G-212

A thin non-conducting rod of length L lies along the z-axis from $z = -L/2$ to $z = L/2$. It carries a total charge Q distributed uniformly along its length.

1. Find the potential ϕ due to the charged rod, for any point on the z axis with $z > L/2$.

2. Find $\phi(r, \theta, \phi)$ for all $r > L/2$, where (r, θ, ϕ) are the usual spherical polar coordinates, in the form of a power series expansion.

   Hint: The general solution of Laplace's equation with azimuthal symmetry can be written as

$$\phi = \sum_{l=0}^{\infty} \left(a_l r^l + b_l r^{-l-1}\right) P_l(\cos(\theta))$$

   The Legendre polynomials $P_l(x)$ are normalised so that $P_l(1) = 1$.

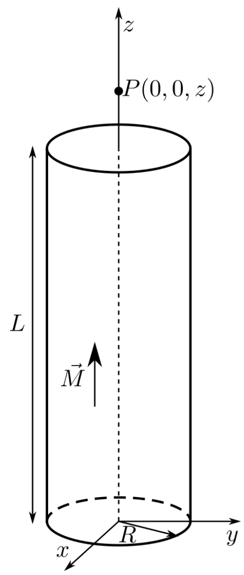
Problem 176. *2000-Fall-EM-G-4* ID:EM-G-215

A cylinder of radius R and length L has a uniform magnetization $\vec{M}$ along its symmetry axis as shown in the figure.

1. Determine the effective volume and surface current densities due to the magnetization.

2. Compute the magnetic field $\vec{B}$ at a point P on the symmetry axis of the cylinder. Check that your answer makes sense if $z = L/2$ and $L \gg R$.

3. Sketch the qualitative behavior of $\vec{B}$ and $\vec{H}$ in the $y - z$ plane.



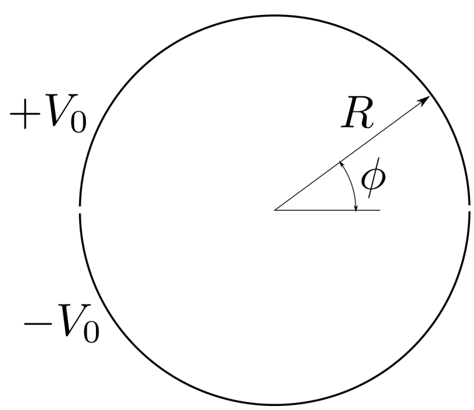
Problem 177. *2000-Fall-EM-G-5* ID:EM-G-218

An infinitely long, thin-walled circular cylinder of radius R is split into two half cylinders. The upper half is fixed at constant potential $V = +V_0$, while the lower half is fixed at the potential $V = -V_0$ as shown in the figure. The two halves are separated by an infinitesimal gap.

1. 2) What is the symmetry property of the electric potential Φ under the transformation $\phi \to -\phi$?

2. Find the potential inside and outside the cylinder.

3. Calculate the charge density as a function of ϕ. (If your answer involves an infinite sum, you need not calculate this sum.)



Problem 178. *2000-Spring-EM-G-4* ID:EM-G-221

A uniformly charged disk of radius a, with charge σ per unit area. lies in the $x - y$ plane, centered on the origin.

1. Calculate the electrostatic potential $\Phi(z)$ everywhere on the z-axis.

2. Expand the resulting $\Phi(z)$ in a power series valid in the region $|z| > a$, including at least the first three non-vanishing terms. Use this expansion to obtain the solution for $\Phi(r, \theta)$ in the region $r > a$, $0 \leq \theta \leq \pi$, in spherical polar coordinates (r, θ, ϕ), in the form of an expansion involving $P_l(\cos(\theta))$ to at least $l = 4$. (Recall that $P_l(1) = 1$ for all l.)

Problem 179. *2001-Fall-EM-G-4* ID:EM-G-227

1. Calculate the electric field $\vec{E}$ due to a dipole $\vec{p}$, whose electrostatic potential is $\phi = \vec{p} \cdot \vec{r}/r^3$.

2. The dipole is placed at a height h above a perfectly conducting infinite plane, and makes an angle θ with respect to the normal to the plane. Draw the position and orientation of the dipole image, and indicate the direction of the force experienced by the dipole.

3. Calculate the work required to remove the dipole to infinity.

Problem 180. *2001-Fall-EM-G-5* ID:EM-G-230

A cylindrical shell of inner radius a and outer radius b has its axis of symmetry along the z-axis; it is made of iron with permeability constant μ. The shell is in a region with uniform external magnetic flux $\vec{B}_0 = B_0\hat{x}$. Using cylindrical coordinates (r, θ, ϕ), let 1, 2, and 3 label the regions $r < a$, $a < r < b$, and $b < r$, respectively; let the corresponding magnetic scalar potentials be ϕ_1, ϕ_2, and ϕ_3.

1. Write down the boundary conditions obeyed by ϕ_1, ϕ_2, and ϕ_3.

2. Assuming that $\mu \gg \mu_0$, compute ϕ_1.

3. Compute the magnetic field, $\vec{H}_1$, in region 1 for $\mu \gg \mu_0$ to leading nontrivial order in μ/μ_0. Sketch the magnetic field lines.

Problem 181. *2001-Spring-EM-G-4* ID:EM-G-233

Consider the vector potential for a magnetized material with magnetic dipole moment per unit volume $\vec{M}$:

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{x}') \times (\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3} d^3x'.$$

1. Show how $\vec{A}$ can be computed in terms of an equivalent volume current density $\vec{J}_b$ in the material and an equivalent surface current density $\vec{K}_s$ on the boundary of the material. Specify the definitions of $\vec{J}_b$ and $\vec{K}_s$ in terms of $\vec{M}$.

2. Let a solid sphere of radius R have uniform magnetization that points in the positive z-direction, so $\vec{M} = M_0\hat{z}$, where M_0 is a constant. Find the equivalent current densities $\vec{J}_b$ and $\vec{K}_s$. Using $\vec{J}_b$ and $\vec{K}_s$ compute the vector potential $\vec{A}$ and the magnetic field $\vec{B}$ at an arbitrary point outside the sphere.

Note: You may find some of the following formulae useful:

$$\vec{\nabla} \times (f\vec{A}) = f(\vec{\nabla} \times \vec{A}) - \vec{A} \times (\nabla f), \qquad P_l(\cos\gamma) = \frac{4p}{2l+1}\sum_{m=-l}^{l} Y^*_{lm}(\theta', \phi')Y_{lm}(\theta, \phi)$$

$$Y_{00} = \tfrac{1}{\sqrt{4\pi}}, \qquad Y_{11} = -\sqrt{\frac{3}{8\pi}}\sin(\theta)e^{i\phi},$$

$$Y_{10} = \sqrt{\tfrac{3}{8\pi}}\cos(\theta), \qquad Y_{22} = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2(\theta)e^{2i\phi}$$

$$Y_{21} = -\sqrt{\tfrac{15}{8\pi}}\sin(\theta)\cos(\theta)e^{i\phi}, \qquad Y_{11} = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2(\theta) - \frac{1}{2}\right).$$

Problem 182. *2001-Spring-EM-G-5* ID:EM-G-236

It can be shown that the total electromagnetic force acting on charges in volume V is given by

$$F_i = \oint_{\mathcal{S}} \sum_{j=1}^{3} T_{ij} da_j,$$

where the surface $\mathcal{S}$ is the boundary of the volume V, $d\vec{a}$ is the outward-pointing infinitesimal surface area element and T_{ij} is the Maxwell stress tensor given by

$$T_{ij} = \epsilon_0\left(E_iE_j - \frac{1}{2}\delta_{ij}E^2\right) + \frac{1}{\mu_0}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right).$$

1. Consider a uniformly charged sphere of radius R and total charge Q. Compute T_{zx}, T_{zy}, and T_{zz} on the surface of the upper hemisphere and on the surface of the equatorial disc that separates the upper and lower hemisphere.

2. Using these results, compute the net force the lower hemisphere exerts on the upper hemisphere.